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Dispersion of a Solute in Peristaltic Motion of a Couple Stress Fluid in the Presence of Magnetic Field

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Abstract—An analytical solution for dispersion of a solute in the peristaltic motion of a couple stress fluid in the presence of magnetic field with both homogeneous and heterogeneous chemical reactions is presented. The average effective dispersion coefficient has been found using Taylor's limiting condition and long wavelength approximation. The effects of various relevant parameters on the average effective coefficient of dispersion have been studied. The average effective dispersion coefficient tends to decrease with magnetic field parameter, homogeneous chemical reaction rate parameter and amplitude ratio but tends to increase with heterogeneous chemical reaction rate parameter.

Keywords—Dispersion, Peristalsis, Couple stress fluid, Chemical reaction, Magnetic field.

I. Introduction

THE process of dispersion of a solute in fluids flowing through channels or pipes has been extensively investigated because of its important applications in various chemical and biological systems. The study of such problem was initiated by Taylor [1]–[3], who presented an analysis to discuss dispersion of a soluble salt when ejected to a stream of solvent flowing slowly through a tube. This analysis was later generalized and extended by many researchers to study dispersion of solute in Newtonian or non-Newtonian fluid flows under various situations (Aris [4], Dutta et al. [5], Shukla et al. [6], Chandra and Agarwal [7], Philip and Chandra [8] and Soundalgekar and Chaturani [9]). The effects of homogeneous and/or heterogeneous chemical reactions on the dispersion of a solute have also been studied by numerous authors under different conditions (Gupta and Gupta [10], Ramana Rao and Padma [11], [12], Padma and Ramana Rao [13], Shukla et al. [6], and Philip and Peeyush [8]).

Peristalsis is a natural mechanism of transport for many physiological fluids. This is achieved by the passage of progressive waves of area contraction or expansion along the boundary of a fluid-filled distensible tube. Different physiological phenomena, such as the flow of urine from kidney to the bladder through ureters, transport of food material through the digestive tract, movement of spermatozoa in the ductus efferentes of the male reproductive tract and cervical canal and the transport of ovum in the fallopian tube, take place by the mechanism of peristalsis. Some biomedical instruments such as blood pumps in dialysis and the heart lung machine use this principle. Peristaltic transport of a toxic liquid is used in nuclear industry to avoid contamination of the outside environment. The industrial use of this pumping mechanism in roller/finger pumps to pump slurries and corrosive fluids is well known. Several studies have been made on peristalsis with reference to mechanical and physiological situations. (Shapiro et al. [14], Fung and Yih [15], Misra and Pandey [16], [17], Mishra and Rao [18], Radhakrishnamacharya [19]).

Magnetohydrodynamics (MHD) is the science which deals with the motion of conducting fluids in the presence of a magnetic field. The motion of the conducting fluid across the magnetic field generates electric currents which change the magnetic field and the action of the magnetic field on these currents gives rise to mechanical forces which modify the flow of the fluid (Mekheimer [20]). MHD flow of a fluid in a channel with elastic, rhythmically contracting walls (peristaltic flow) is of interest in connection with certain problems of the movement of conductive physiological fluids (example: the blood and blood pump machines) (Hayat et al. [21]). Currently, studies on peristaltic motion in MHD flows of electrically conducting physiological fluids have become a subject of growing interest for researchers. This is due to the fact that such studies are useful particularly for getting a proper understanding of the functioning of different machines used by clinicians for pumping blood (Misra et al. [22]). Misra et al. [22] pointed out that theoretical researches with an aim to explore the effect of a magnetic field on the flow of blood in atherosclerotic vessels also find application in a blood pump used by cardiac surgeons during the surgical procedure.

It is well known that most physiological fluids including blood behave as non-Newtonian fluids. Hence, the study of peristaltic transport of non-Newtonian fluids may help to get better understanding of the biological systems. Several researchers studied peristaltic transport of non-Newtonian fluids (Radhakrishnamacharya [19], Ramachandra Rao and Mishra [23], Misra and Pandey [16] and Hayat et al. [21]).

Couple stress fluids are fluids consisting of rigid, randomly oriented particles suspended in a viscous medium. Couple stress fluid is known to be a better model for bio-fluids, such as blood, lubricants containing small amount of high polymer additive, electro-rheological fluids and synthetic fluids. The main feature of couple stress fluids is that the stress tensor is anti-symmetric and their accurate flow behavior cannot be predicted by the classical Newtonian theory. Stokes [24] generalized the classical model to include the effect of the presence of the couple stresses and this model has been widely used because of its relative mathematical simplicity (Islam

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and Zhou [25]). For couple stress fluids, there have been a number of studies carried out due to its widespread industrial and scientific applications, such as the works of Stokes [24], Srivastava [26], Mekheimer and Abd elmaboud [27] and Sobh [28].

Dispersion of a solute in peristaltic motion of a couple stress fluid in the presence of magnetic field has not received much attention. It is realized that magnetic field and peristalsis may have significant effect on the dispersion of a solute in the flow of conducting fluid and this may lead to better understanding of the flow situation in physiological systems. The objective of this paper is to study the dispersion of a solute in peristaltic motion of a couple stress fluid in the presence of magnetic field. Using long wavelength approximation and Taylor's approach, closed form solution has been obtained for the dispersion coefficient for both the cases of homogeneous first-order irreversible chemical reaction and combined first-order homogeneous and heterogeneous chemical reactions. The effects of various relevant parameters on the average effective dispersion coefficient are studied.

II. Mathematical Formulation

Consider dispersion of a solute in peristaltic flow of an electrically conducting couple stress fluid in an infinite uniform channel of width $2d$ and with flexible walls on which traveling sinusoidal waves of long wavelength are imposed. A uniform magnetic field B_0 is applied to the fluid normal to the walls of the channel. Cartesian coordinate system (x, y) is chosen with the x-axis aligned with the center line of the channel. The traveling waves are represented by (Fig.1)

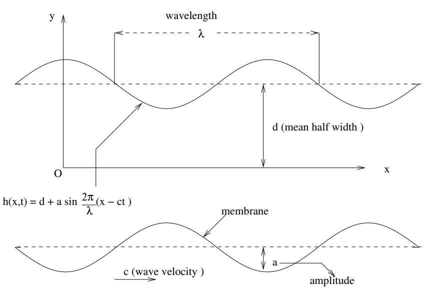

Fig. 1. Geometry of the problem.

$$y = \pm h = \pm \left[d + a \sin \frac{2\pi}{\lambda}(x - ct) \right] \qquad (1)$$

where a is the amplitude, c is the wave speed and λ is the wavelength of the peristaltic wave.

Under long wavelength approximation and neglecting body forces and body couples, the equations governing the peristaltic motion of incompressible couple stress fluid in the presence of magnetic field for the present problem are given as

$$\begin{aligned} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 && (2)\\ -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - \eta' \frac{\partial^4 u}{\partial y^4} - \sigma B_0^2 u &= 0 && (3)\\ -\frac{\partial p}{\partial y} &= 0 && (4) \end{aligned}$$

where $u(x, y, t)$ and $v(x, y, t)$ are the velocity components in the x and y directions respectively, p is the pressure, μ is the viscosity coefficient of classical fluid dynamics, η' is the couple stress fluid viscosity , σ is the electrical conductivity of the fluid and B_0 is the uniform magnetic field.

We assume that the walls are inextensible so that only lateral motion takes place and the horizontal displacement of the wall is zero.Thus, the relevant boundary conditions for the velocity are given by

$$u = 0, \qquad \frac{\partial^2 u}{\partial y^2} = 0 \qquad at \qquad y = \pm h \qquad (5)$$

Solving (2)-(4) under the boundary conditions (5), the velocity is given as

$$u(y) = -\frac{1}{\sigma B_0^2} \frac{\partial p}{\partial x} \left[S_2 \cosh(m_1^* y) - S_1 \cosh(m_2^* y) + 1 \right] \qquad (6)$$

where

$$\begin{aligned} m_1^* &= \sqrt{(\mu/2\eta') \left(1 + \sqrt{1 - 4\sigma B_0^2 \eta'/\mu^2} \right)},\\ m_2^* &= \sqrt{(\mu/2\eta') \left(1 - \sqrt{1 - 4\sigma B_0^2 \eta'/\mu^2} \right)},\\ S_1 &= \frac{(m_1^*)^2}{\cosh(m_2^* h) \left[(m_1^*)^2 - (m_2^*)^2 \right]} \quad and\\ S_2 &= \frac{(m_2^*)^2}{\cosh(m_1^* h) \left[(m_1^*)^2 - (m_2^*)^2 \right]}. \end{aligned}$$

Further, the mean velocity is defined as

$$\bar{u} = \frac{1}{2h} \int_{-h}^{+h} u(y) dy. \qquad (7)$$

Substituting (6) in (7) we get,

$$\bar{u} = -\frac{1}{\sigma B_0^2} \frac{\partial p}{\partial x} \left[\frac{S_2}{m_1^* h} \sinh(m_1^* h) - \frac{S_1}{m_2^* h} \sinh(m_2^* h) + 1 \right]. \qquad (8)$$

If we now consider convection across a plane moving with the mean speed of the flow, then relative to this plane, the fluid velocity is given by

$$\begin{aligned} u_x &= u - \bar{u}\\ &= -\frac{1}{\sigma B_0^2} \frac{\partial p}{\partial x} \left[S_2 \cosh(m_1^* y) - S_1 \cosh(m_2^* y) \right.\\ &- \left. \frac{S_2}{m_1^* h} \sinh(m_1^* h) + \frac{S_1}{m_2^* h} \sinh(m_2^* h) \right]. \qquad (9) \end{aligned}$$

A. Diffusion with a Homogeneous First-order Chemical Reaction

It is assumed that a solute diffuses and simultaneously undergoes a first order irreversible chemical reaction in peristaltic transport of a couple stress fluid in a channel. Assuming isothermal conditions and $\frac{\partial^2 C}{\partial x^2} << \frac{\partial^2 C}{\partial y^2}$ (Gupta and Gupta [10]), the equation for the concentration C of the solute for the present problem satisfies the diffusion equation

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) - k_1 C \qquad (10)$$

where D is the molecular diffusion coefficient and k_1 is the first order reaction rate constant. For typical values of physiologically relevant parameters of this problem, it is realized that $\bar{u} \approx c$. Using this condition and following Taylor [1]–[3], we assume partial equilibrium is maintained and then making use of the following dimensionless quantities

$$\theta = t/\bar{t},\ \bar{t} = \lambda/\bar{u},\ \eta = y/d,\ \xi = (x - \bar{u}t)/\lambda,\ H = h/d, \qquad (11)$$

equation (9) reduces to

$$\begin{aligned} u_x &= -\frac{1}{\sigma B_0^2}\frac{\partial p}{\partial x}\left[S_2\cosh(m_1\eta) - S_1\cosh(m_2\eta)\right. \\ &- \left.\frac{S_2\sinh(m_1 H)}{m_1 H} + \frac{S_1\sinh(m_2 H)}{m_2 H}\right]. \end{aligned} \qquad (12)$$

where

$$\begin{aligned} m_1 &= m_1^* d = \sqrt{(m^2/2)\left(1 + \sqrt{1 - 4(H^*)^2/m^2}\right)}, \\ m_2 &= m_2^* d = \sqrt{(m^2/2)\left(1 - \sqrt{1 - 4(H^*)^2/m^2}\right)}, \\ H^* &= B_0 d(\sigma/\mu)^{1/2}, \qquad m = d(\mu/\eta')^{1/2}, \end{aligned}$$

m is the couple stress parameter and H^* is the Hartmann number (or magnetic field parameter).

Further, (10) becomes

$$\frac{\partial^2 C}{\partial \eta^2} - \frac{k_1 d^2}{D}C = \frac{d^2}{\lambda D}u_x\frac{\partial C}{\partial \xi}. \qquad (13)$$

Assuming that there is no absorption at the walls, the boundary conditions for the concentration C are

$$\frac{\partial C}{\partial \eta} = 0 \qquad for \qquad \eta = \pm H = \pm[1 + \epsilon\sin(2\pi\xi)] \quad (14)$$

where $\epsilon = a/d$ is the amplitude ratio.

Assuming that $\partial C/\partial \xi$ is independent of η at any cross-section and solving (13) under the boundary conditions (14), the solution for the concentration of the solute C is given as

$$\begin{aligned} C(\eta) = A\cosh(\alpha\eta) - \frac{d^2}{\lambda D}\frac{1}{\sigma B_0^2}\frac{\partial C}{\partial \xi}\frac{\partial p}{\partial x}\left\{\frac{S_2\cosh(m_1\eta)}{m_1^2 - \alpha^2}\right. \\ - \left.\frac{S_1\cosh(m_2\eta)}{m_2^2 - \alpha^2} + \frac{S_2\sinh(m_1 H)}{\alpha^2 m_1 H} - \frac{S_1\sinh(m_2 H)}{\alpha^2 m_2 H}\right\} \end{aligned} \qquad (15)$$

where

$$\begin{aligned} A = \frac{d^2}{\lambda D}\frac{\partial C}{\partial \xi}\frac{1}{\sigma B_0^2}\frac{\partial p}{\partial x}\frac{1}{L}\left[\frac{m_1 S_2}{m_1^2 - \alpha^2}\sinh(m_1 H)\right. \\ - \left.\frac{m_2 S_1}{m_2^2 - \alpha^2}\sinh(m_2 H)\right], \end{aligned} \qquad (16)$$

$\alpha = d(k_1/D)^{1/2}$ and $L = \alpha\sinh(\alpha H)$.

The volumetric rate Q at which the solute is transported across a section of the channel of unit breadth is defined by

$$Q = \int_{-H}^{+H} C u_x d\eta. \qquad (17)$$

Substituting (15) and (12) in (17), we get the volumetric rate Q as

$$Q = -\frac{2d^6}{\lambda\mu^2 D}\frac{\partial C}{\partial \xi}\left(\frac{\partial p}{\partial x}\right)^2 F(\xi, \epsilon, \alpha, m, H^*) \qquad (18)$$

where

$$\begin{aligned} &F(\xi, \epsilon, \alpha, m, H^*) = \\ &\frac{1}{(H^*)^4}\left\{\frac{S_2\,\mathrm{csch}(\alpha H)}{\alpha(m_1^2 - \alpha^2)}\left(\frac{m_1 S_2\sinh(m_1 H)}{m_1^2 - \alpha^2} - \frac{m_2 S_1\sinh(m_2 H)}{m_2^2 - \alpha^2}\right)B_1\right. \\ &+\frac{S_1\,\mathrm{csch}(\alpha H)}{\alpha(m_2^2 - \alpha^2)}\left(\frac{m_2 S_1\sinh(m_2 H)}{m_2^2 - \alpha^2} - \frac{m_1 S_2\sinh(m_1 H)}{m_1^2 - \alpha^2}\right)B_2 \\ &+\frac{S_1 S_2}{\alpha^2 H}\sinh(m_1 H)\sinh(m_2 H)\left(\frac{m_1}{m_2(m_1^2 - \alpha^2)} + \frac{m_2}{m_1(m_2^2 - \alpha^2)}\right) \\ &-\frac{S_2^2\sinh^2(m_1 H)}{\alpha^2 m_1^2 H} - \frac{S_1^2\sinh^2(m_2 H)}{\alpha^2 m_2^2 H} - \frac{S_2^2}{m_1^2 - \alpha^2}\left(H + \frac{\sinh(2m_1 H)}{2m_1}\right) \\ &+\frac{B_3 S_1 S_2}{m_1^2 - \alpha^2}\left(\frac{1}{m_1^2 - \alpha^2} + \frac{1}{m_2^2 - \alpha^2}\right) - \frac{S_1^2}{m_2^2 - \alpha^2}\left(H + \frac{\sinh(2m_2 H)}{2m_2}\right) \\ &-\left.\frac{S_1 S_2}{m_1 m_2 H}\sinh(m_1 H)\sinh(m_2 H)\left(\frac{1}{m_1^2 - \alpha^2} + \frac{1}{m_2^2 - \alpha^2}\right)\right\}, \end{aligned} \qquad (19)$$

$$\begin{aligned} B_1 &= m_1\cosh(\alpha H)\sinh(m_1 H) - \alpha\cosh(m_1 H)\sinh(\alpha H), \\ B_2 &= m_2\cosh(\alpha H)\sinh(m_2 H) - \alpha\cosh(m_2 H)\sinh(\alpha H), \\ B_3 &= m_1\cosh(m_2 H)\sinh(m_1 H) - m_2\cosh(m_1 H)\sinh(m_2 H) \end{aligned}$$

Comparing (18) with Fick's law of diffusion, we find that the solute is dispersed relative to a plane moving with the mean speed of the flow with an effective dispersion coefficient D^* given by

$$D^* = 2\frac{d^6}{\mu^2 D}\left(\frac{\partial p}{\partial x}\right)^2 F(\xi, \epsilon, \alpha, m, H^*). \qquad (20)$$

Let the average of F be $\overline{F}$ and is defined by

$$\overline{F} = \int_0^1 F(\xi, \epsilon, \alpha, m, H^*)d\xi. \qquad (21)$$

B. Diffusion with Combined Homogeneous and Heterogeneous Chemical Reactions

We now discuss the problem of diffusion with a first-order irreversible chemical reaction taking place both in the bulk of the medium (homogeneous) as well as at the walls (heterogeneous) of the channel which are assumed to be catalytic to chemical reaction. The diffusion equation is same as given by (10), i.e.,

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}=D\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)-k_1 C.$$

The differential material balance at the walls (Philip and Chandra [8]) gives the boundary conditions

$$\frac{\partial C}{\partial y}+fC=0 \quad at \quad y=h=\left[d+a\sin\frac{2\pi}{\lambda}(x-\bar{u}t)\right], \tag{22}$$

$$\frac{\partial C}{\partial y}-fC=0 \quad at \quad y=-h=-\left[d+a\sin\frac{2\pi}{\lambda}(x-\bar{u}t)\right]. \tag{23}$$

If we introduce the dimensionless variables (11) and assume the limiting condition of Taylor [1]–[3], the diffusion equation remains as (13) but subject to the boundary conditions

$$\frac{\partial C}{\partial \eta}+\beta C=0 \quad for \quad \eta=H=[1+\epsilon\sin(2\pi\xi)], \tag{24}$$

$$\frac{\partial C}{\partial \eta}-\beta C=0 \quad for \quad \eta=-H=-[1+\epsilon\sin(2\pi\xi)], \tag{25}$$

where $\beta=fd$ is the heterogeneous reaction rate parameter corresponding to catalytic reaction at the walls.

The solution of (13) satisfying the boundary conditions (24) and (25) is

$$\begin{aligned}C(\eta)&=A^{'}\cosh(\alpha\eta)+\frac{d^2}{\lambda D}\frac{\partial C}{\partial \xi}\frac{1}{\sigma B_0^2}\frac{\partial p}{\partial x}\\&\times\left[\frac{S_2}{m_1^2-\alpha^2}\cosh(m_1\eta)-\frac{S_1}{m_2^2-\alpha^2}\cosh(m_2\eta)\right.\\&\left.+\frac{S_2}{\alpha^2 m_1 H}\sinh(m_1H)-\frac{S_1}{\alpha^2 m_2 H}\sinh(m_2H)\right]\end{aligned} \tag{26}$$

where

$$\begin{aligned}A^{'}&=\frac{d^2}{\lambda D}\frac{\partial C}{\partial \xi}\frac{1}{\sigma B_0^2}\frac{\partial p}{\partial x}\frac{1}{L^{'}}\left[\frac{m_1S_2}{m_1^2-\alpha^2}\sinh(m_1H)\right.\\&-\frac{m_2S_1}{m_2^2-\alpha^2}\sinh(m_2H)+\frac{\beta S_2}{m_1^2-\alpha^2}\cosh(m_1H)\\&-\frac{\beta S_1}{m_2^2-\alpha^2}\cosh(m_2H)+\frac{\beta S_2}{\alpha^2 m_1 H}\sinh(m_1H)\\&\left.-\frac{\beta S_1}{\alpha^2 m_2 H}\sinh(m_2H)\right]\end{aligned} \tag{27}$$

and $L^{'}=\alpha\sinh(\alpha H)+\beta\cosh(\alpha H)$.

Substituting (26) and (12) in (17), we get

$$Q=-2\frac{d^6}{\lambda\mu^2 D}\frac{\partial C}{\partial \xi}\left(\frac{\partial p}{\partial x}\right)^2 G(\xi,\epsilon,\alpha,\beta,m,H^*) \tag{28}$$

where

$$\begin{aligned}&G(\xi,\epsilon,\alpha,\beta,m,H^*)=\frac{1}{(H^*)^4}\\&\times\left\{\frac{S_2}{L'(m_1^2-\alpha^2)}\left(\frac{m_1S_2\sinh(m_1H)}{m_1^2-\alpha^2}-\frac{m_2S_1\sinh(m_2H)}{m_2^2-\alpha^2}\right)B_1\right.\\&+\frac{S_1}{L'(m_2^2-\alpha^2)}\left(\frac{m_2S_1\sinh(m_2H)}{m_2^2-\alpha^2}-\frac{m_1S_2\sinh(m_1H)}{m_1^2-\alpha^2}\right)B_2\\&-\frac{\sinh(\alpha H)}{\alpha HL'}\left(\frac{S_2^2\sinh^2(m_1H)}{m_1^2-\alpha^2}+\frac{S_1^2\sinh^2(m_2H)}{m_2^2-\alpha^2}\right)\\&+\frac{S_1S_2}{\alpha HL'}\left(\frac{m_1\sinh(\alpha H)}{m_2(m_1^2-\alpha^2)}+\frac{m_2\sinh(\alpha H)}{m_1(m_2^2-\alpha^2)}\right)\sinh(m_1H)\sinh(m_2H)\\&+\frac{\beta S_2}{L'(m_1^2-\alpha^2)}\left(\frac{S_2\cosh(m_1H)}{m_1^2-\alpha^2}-\frac{S_1\cosh(m_2H)}{m_2^2-\alpha^2}\right)B_1\\&+\frac{\beta S_1}{L'(m_2^2-\alpha^2)}\left(\frac{S_1\cosh(m_2H)}{m_2^2-\alpha^2}-\frac{S_2\cosh(m_1H)}{m_1^2-\alpha^2}\right)B_2\\&-\frac{\beta}{\alpha HL'}\left(\frac{S_2^2\cosh(m_1H)\sinh(m_1H)}{m_1(m_1^2-\alpha^2)\operatorname{csch}(\alpha H)}+\frac{S_1^2\cosh(m_2H)\sinh(m_2H)}{m_2(m_2^2-\alpha^2)\operatorname{csch}(\alpha H)}\right)\\&+\frac{\beta S_1S_2}{\alpha HL'}\left(\frac{\cosh(m_1H)\sinh(m_2H)}{m_2(m_1^2-\alpha^2)\operatorname{csch}(\alpha H)}+\frac{\cosh(m_2H)\sinh(m_1H)}{m_1(m_2^2-\alpha^2)\operatorname{csch}(\alpha H)}\right)\\&+\frac{\beta S_2}{\alpha^2HL'(m_1^2-\alpha^2)}\left(\frac{S_2\sinh(m_1H)}{m_1}-\frac{S_1\sinh(m_2H)}{m_2}\right)B_1\\&+\frac{\beta S_1}{\alpha^2HL'(m_2^2-\alpha^2)}\left(\frac{S_1\sinh(m_2H)}{m_2}-\frac{S_2\sinh(m_1H)}{m_1}\right)B_2\\&-\frac{\beta\sinh(\alpha H)}{\alpha^3H^2L'}\left(\frac{S_2^2\sinh^2(m_1H)}{m_1^2}+\frac{S_1^2\sinh^2(m_2H)}{m_2^2}\right)\\&+\frac{2\beta S_1S_2\sinh(m_1H)\sinh(m_2H)}{\alpha^3m_1m_2H^2L'\operatorname{csch}(\alpha H)}-\frac{S_2^2}{m_1^2-\alpha^2}\left(H+\frac{\sinh(2m_1H)}{2m_1}\right)\\&+\frac{S_1S_2}{m_1^2-\alpha^2}\left(\frac{1}{m_1^2-\alpha^2}+\frac{1}{m_2^2-\alpha^2}\right)B_3+\frac{S_2^2\sinh^2(m_1H)}{m_1^2H(m_1^2-\alpha^2)}\\&-\frac{S_1S_2}{m_1m_2H}\sinh(m_1H)\sinh(m_2H)\left(\frac{1}{m_1^2-\alpha^2}+\frac{1}{m_2^2-\alpha^2}\right)\\&\left.-\frac{S_1^2}{m_2^2-\alpha^2}\left(H+\frac{\sinh(2m_2H)}{2m_2}\right)+\frac{S_1^2\sinh^2(m_2H)}{m_2^2H(m_2^2-\alpha^2)}\right\}.\end{aligned} \tag{29}$$

Comparing (28) with Fick's Law of Diffusion, we find that the solute is dispersed relative to a plane moving with the mean speed of the flow with an effective dispersion coefficient D^* given by

$$D^*=2\frac{d^6}{\mu^2 D}\left(\frac{\partial p}{\partial x}\right)^2 G(\xi,\epsilon,\alpha,\beta,m,H^*). \tag{30}$$

The average of G denoted by $\overline{G}$ is defined as

$$\overline{G}=\int_0^1 G(\xi,\epsilon,\alpha,\beta,m,H^*)d\xi. \tag{31}$$

III. Results and Discussion

As given in (21)and (31), the expressions for $\overline{F}$ and $\overline{G}$ have been obtained by numerical integration using MATHEMATICA software. The effects of various parameters on the average effective dispersion coefficient can be observed through the functions $\overline{F}(\xi,\epsilon,\alpha,m,H^*)$ (for homogeneous case) and $\overline{G}(\xi,\epsilon,\alpha,\beta,m,H^*)$ (for combined homogeneous and heterogeneous case). The functions $\overline{F}$ and $\overline{G}$ have been numerically evaluated for different values of relevant parameters and presented graphically. The important parameters involved in the expressions are: the amplitude ratio ϵ, the homogeneous reaction rate parameter α, the heterogeneous reaction rate parameter β, the magnetic field parameter (or Hartmann number) H^*, and couple stress parameter m.

A. Homogeneous Chemical Reaction

Figs. 2-4 show that average effective dispersion coefficient $\overline{F}$ decreases with homogeneous reaction rate parameter α. This implies that homogeneous chemical reaction tends to decrease the dispersion of the solute. This result is expected since increase in α leads to increasing number of moles of solute undergoing chemical reaction, which results in the decrease of dispersion. The result that dispersion decreases with α agrees with previous results obtained by Gupta and Gupta [10], Dutta et al. [5], Ramana Rao and Padma [11], [12], Padma and Ramana Rao [13], Shukla et al. [6]. Further, average dispersion decreases with magnetic field parameter (or Hartmann number) H^* (Fig.2), couple stress parameter m (Fig. 3) and amplitude ratio ϵ (Fig.4). The result that dispersion decreases with couple stress parameter m agrees with the result obtained by Soundalgekar and Chaturani [9].

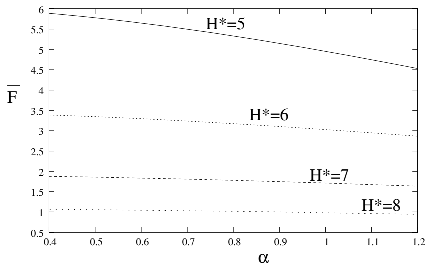

Fig. 2. Effect of H^* on $\overline{F}$ for $m = 5.0$ and $\epsilon = 0.2$.

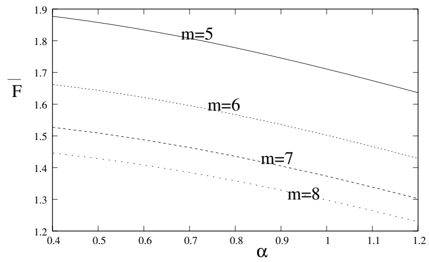

Fig. 3. Effect of m on $\overline{F}$ for $\epsilon = 0.2$ and $H^* = 7.0$.

B. Combined Homogeneous and Heterogeneous Chemical Reactions

Figs. 5-8 show the effects of various parameters on the average dispersion coefficient $\overline{G}$ for the case of combined first order chemical reactions both in the bulk and at the walls. Average dispersion coefficient $\overline{G}$ decreases with magnetic field

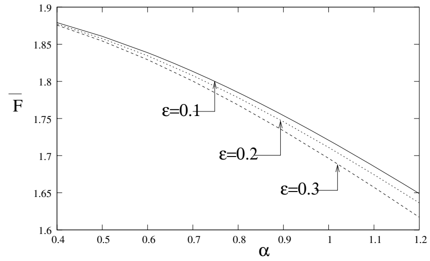

Fig. 4. Effect of ϵ on $\overline{F}$ for $m = 5.0$ and $H^* = 7.0$.

parameter H^* (Fig.5), amplitude ratio ϵ (Fig.7) and homogeneous chemical reaction parameter α (Fig.8) but increases with couple stress parameter m (Fig.6). The decrease with amplitude ratio ϵ is less significant for lower heterogeneous chemical reaction rate ($\beta \leq 2$) (Fig.7). Further, Figs. 5-8 show that average dispersion increases with heterogeneous reaction rate parameter β.

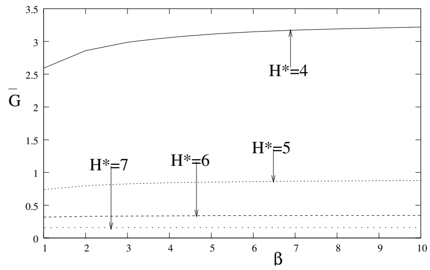

Fig. 5. Effect of H^* on $\overline{G}$ for $\alpha = 1.0$, $m = 6.0$ and $\epsilon = 0.2$.

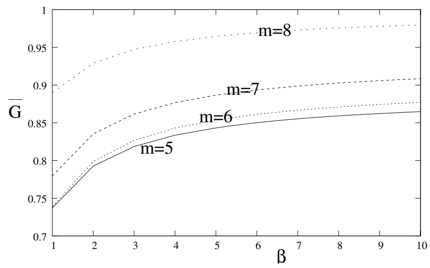

Fig. 6. Effect of m on $\overline{G}$ for $\alpha = 1.0$, $H^* = 5.0$ and $\epsilon = 0.2$.

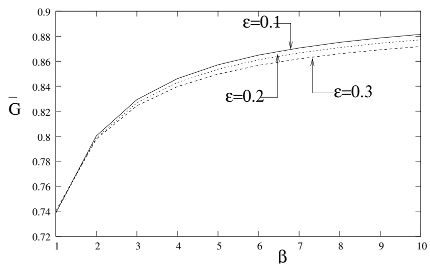

Fig. 7. Effect of ϵ on $\overline{G}$ for $m = 6.0$, $H^* = 5.0$ and $\alpha = 1.0$.

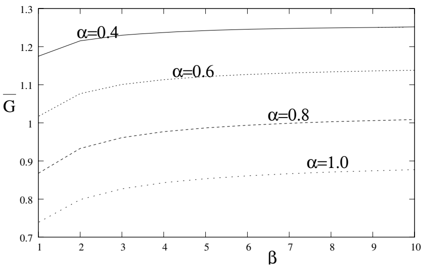

Fig. 8. Effect of α on $\overline{G}$ for $m = 6.0$, $H^* = 5.0$ and $\epsilon = 0.2$.

IV. Conclusion

The dispersion of a solute in peristaltic motion of a couple stress fluid in the presence of magnetic field with both homogeneous and heterogeneous chemical reactions has been studied under long wavelength approximation and Taylor's limiting condition. It is observed that average effective coefficient of dispersion decreases with magnetic field parameter (or Hartmann number) H^*, homogeneous chemical reaction rate parameter α and amplitude ratio ϵ in both the cases. Further, dispersion decreases with couple stress parameter m in the case of homogeneous chemical reaction but increases with it in the case of combined homogeneous and heterogeneous chemical reactions. Dispersion also tends to increase with heterogeneous chemical reaction rate parameter β.

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