

# Spectrum Sensing Based On the Cyclostationarity of PU Signals in High Traffic Environments

Keunhong Chae, Youngpo Lee, and Seokho Yoon

**Abstract**—In cognitive radio (CR) systems, the primary user (PU) signal would randomly depart or arrive during the sensing period of a CR user, which is referred to as the high traffic environment. In this paper, we propose a novel spectrum sensing scheme based on the cyclostationarity of PU signals in high traffic environments. Specifically, we obtain a test statistic by applying an estimate of spectral autocohereance function of the PU signal to the generalized likelihood ratio. From numerical results, it is confirmed that the proposed scheme provides a better spectrum sensing performance compared with the conventional spectrum sensing scheme based on the energy of the PU signals in high traffic environments.

**Keywords**—Spectrum sensing, cyclostationarity, high traffic environments.

## I. INTRODUCTION

AS the recent wireless applications occupy wide frequency bands, the frequency spectrum is increasingly becoming scarce, and thus, efficient use of the spectrum resource is required. The cognitive radio (CR) is a promising technology to exploit underutilized spectrum in an opportunistic manner and the spectrum sensing technique identifying spectrum opportunities is one of the most important techniques in CR [1], [2].

Conventionally, the spectrum sensing techniques have been developed under low traffic environments where the spectrum band is assumed to be occupied by the primary user (PU) or to be vacant during the whole sensing period [3], [4]. Practically, however, the PU signal may depart or arrive during the sensing period, especially when a long sensing period is used to achieve good sensing performance, or when spectrum sensing is performed for a high traffic network, and under such high traffic environments, the performances of the conventional spectrum sensing techniques have been found to degrade severely [5]. Although a spectrum sensing technique [6] was proposed based on the energy detection approach for high traffic environments, it performs poorly when the signal-to-noise ratio (SNR) is low.

In this paper, a novel spectrum sensing scheme is proposed based on the cyclostationarity of PU signals in high traffic environments. We first formulate the spectrum sensing problem in high traffic environments as a binary hypothesis testing problem, and then, obtain a test statistic by applying an estimate of spectral autocohereance function (SAF) of the PU signal to generalized likelihood ratio (GLR). The proposed cyclostationarity-based scheme is expected to provide a better spectrum sensing performance than the conventional energy

detection-based scheme in [6], since the detection performance of the cyclostationarity approach is generally better than that of the energy detection approach, and also, able to distinguish the PU signal from the interference unlike the energy detection approach.

The rest of this paper is organized as follows. In Section II, we model the spectrum sensing problem in high traffic environments as a binary hypothesis testing problem. In Section III, we develop a GLR based on the binary hypothesis model, estimate the SAF of the PU signal, and propose a test statistic for spectrum sensing by applying the estimate of the SAF to the GLR. Section IV compares the spectrum sensing performances of the proposed and conventional schemes in terms of the receiver operating characteristic (ROC). Finally, Section V concludes this paper.

## II. SYSTEM MODEL

We model the spectrum sensing problem in high traffic environments where the PU randomly departs or arrives during the sensing period of CR user as a binary hypothesis testing problem: Given the received signal, a decision is to be made between the null hypothesis  $H_0$  and the alternative hypothesis  $H_1$  defined as

$$H_0 : y[n] = \begin{cases} x[n] + w[n], & \text{for } n = 1, 2, \dots, J_0, \\ w[n], & \text{for } n = J_0 + 1, J_0 + 2, \dots, N, \end{cases} \quad (1)$$

and

$$H_1 : y[n] = \begin{cases} w[n], & \text{for } n = 1, 2, \dots, J_1, \\ x[n] + w[n], & \text{for } n = J_1 + 1, J_1 + 2, \dots, N, \end{cases} \quad (2)$$

respectively, where  $y[n]$  and  $x[n]$  represent the  $n$ th sample of the baseband equivalent of the received and PU signals, respectively,  $w[n]$  represents the  $n$ th sample of an additive white Gaussian noise (AWGN) with mean zero and power spectral density (PSD)  $N_0/2$ , and  $N$  is the number of samples available during the sensing period. Under the hypothesis  $H_0$ , the random departure of the PU occurs between the  $J_0$ th and  $(J_0 + 1)$ th samples, on the other hand, under the hypothesis  $H_1$ , the random arrival of the PU occurs between the  $J_1$ th and  $(J_1 + 1)$ th samples. Once a test statistic is obtained for spectrum sensing, the test statistic is compared with a predetermined threshold. If the test statistic exceeds the threshold, the CR user chooses the hypothesis  $H_1$  deciding that the spectrum band is occupied by the PU; otherwise, the CR user chooses the hypothesis  $H_0$  and utilizes the spectrum band.

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### III. PROPOSED SCHEME

Applying the GLR test to the binary hypothesis model of (1) and (2) gives the following test statistic

$$\sum_{n=J_0+1}^N y^2[n] - \sum_{n=1}^{J_1} y^2[n] \underset{H_0}{\overset{H_1}{\geq}} \gamma', \quad (3)$$

where  $\gamma'$  is the threshold determined from a given false alarm probability (i.e.,  $\Pr(H_1|H_0)$ ). To exploit the cyclostationarity of the received PU signal, in (3), we replace  $y[n]$  with the SAF  $\rho_y^\alpha(f)$  defined as [7]

$$\rho_y^\alpha(f) = \frac{S_y^\alpha(f)}{[S_y(f + \alpha/2)S_y(f - \alpha/2)]^{1/2}}, \quad (4)$$

where  $\alpha$  is a cyclic frequency,  $S_y(f)$  is the PSD of  $y(t)$ , and

$$S_y^\alpha(f) = \int_{-\infty}^{\infty} E \left[ y \left( t + \frac{\tau}{2} \right) y^* \left( t - \frac{\tau}{2} \right) e^{-j2\pi\alpha t} \right] e^{-j2\pi f \tau} d\tau \quad (5)$$

is the spectral correlation density (SCD) function with  $(\cdot)^*$  the conjugation operation. From (4) and (5), we can see that the SAF is the normalized version of the SCD.

Since  $\rho_y^\alpha(f)$  is the SAF of a continuous signal  $y(t)$ , we cannot replace the discrete value  $y^2[n]$  of (3) with  $(\rho_y^\alpha(f))^2$  directly. Thus, we employ the discrete estimate  $|\hat{\rho}_y^\alpha(f)|^2$  of the squared magnitude of the SAF obtained as

$$|\hat{\rho}_y^\alpha(f)|^2 = \frac{\left| \sum_{n=1}^N u[n]v^*[n] \right|^2}{\sum_{n=1}^N |u[n]|^2 \sum_{n=1}^N |v[n]|^2} \quad (6)$$

to replace  $y^2[n]$  of (3), where  $u[n] = y[n]e^{j\pi(f-\alpha/2)n}$  and  $v[n] = y[n]e^{j\pi(f+\alpha/2)n}$  are the frequency-shifted versions of  $y[n]$  and its crosscorrelation used in (6) can be obtained as depicted in Figure 1. Now, replacing  $y^2[n]$  with (6) yields

$$\sum_{n=J_0+1}^N |\hat{\rho}_y^\alpha(f)|^2 - \sum_{n=1}^{J_1} |\hat{\rho}_y^\alpha(f)|^2 \underset{H_0}{\overset{H_1}{\geq}} \gamma, \quad (7)$$

where  $\gamma$  is a threshold for the test statistic (7). It should be noted that the values of  $J_0$  and  $J_1$  change randomly depending on the behavior of the PU, and thus, the test statistics unconditional for random departure and arrival are obtained by taking the expectation over (7) with respect to  $J_0$  and  $J_1$ , respectively. Generally, the number of events occurring randomly over a period of time is well modeled by Poisson process [8], and thus, we have

$$\Pr\{J_0\} = [1 - e^{-\lambda_d T}] \cdot [e^{-\lambda_d T}]^{J_0} \quad (8)$$

and

$$\Pr\{J_1\} = [1 - e^{-\lambda_a T}] \cdot [e^{-\lambda_a T}]^{J_1}, \quad (9)$$

where  $\lambda_d$ ,  $\lambda_a$ , and  $T$  represent the departure rate, arrival rate, and sampling interval, respectively. Using (8) and (9), finally,

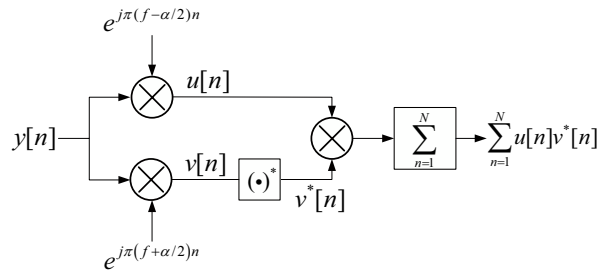


Fig. 1. The crosscorrelation of  $u[n]$  and  $v[n]$ .

we obtain the unconditional test statistics

$$T_d = \frac{\left| \sum_{J_0=0}^{N-1} [1 - e^{-\lambda_d T}] \cdot [e^{-\lambda_d T}]^{J_0} \sum_{n=J_0+1}^N u[n]v^*[n] \right|^2}{\left[ \sum_{J_0=0}^{N-1} [1 - e^{-\lambda_d T}] \cdot [e^{-\lambda_d T}]^{J_0} \sum_{n=J_0+1}^N |u[n]|^2 \right] \left[ \sum_{J_0=0}^{N-1} [1 - e^{-\lambda_d T}] \cdot [e^{-\lambda_d T}]^{J_0} \sum_{n=J_0+1}^N |v[n]|^2 \right]} = \frac{\left| \sum_{n=1}^N [1 - e^{-\lambda_d T n}] u[n]v^*[n] \right|^2}{\sum_{n=1}^N [1 - e^{-\lambda_d T n}] |u[n]|^2 \sum_{n=1}^N [1 - e^{-\lambda_d T n}] |v[n]|^2} \quad (10)$$

$$T_d = \frac{\left| \sum_{J_0=0}^{N-1} [1 - e^{-\lambda_d T}] \cdot [e^{-\lambda_d T}]^{J_0} \sum_{n=J_0+1}^N u[n]v^*[n] \right|^2}{\left[ \sum_{J_0=0}^{N-1} [1 - e^{-\lambda_d T}] \cdot [e^{-\lambda_d T}]^{J_0} \sum_{n=J_0+1}^N |u[n]|^2 \right] \left[ \sum_{J_0=0}^{N-1} [1 - e^{-\lambda_d T}] \cdot [e^{-\lambda_d T}]^{J_0} \sum_{n=J_0+1}^N |v[n]|^2 \right]} = \frac{\left| \sum_{n=1}^N [1 - e^{-\lambda_d T n}] u[n]v^*[n] \right|^2}{\sum_{n=1}^N [1 - e^{-\lambda_d T n}] |u[n]|^2 \sum_{n=1}^N [1 - e^{-\lambda_d T n}] |v[n]|^2} \quad (11)$$

for the random departure of the PU, and similarly,

$$T_a = \frac{\left| \sum_{n=1}^N [1 - e^{-\lambda_a T n}] u[n]v^*[n] \right|^2}{\sum_{n=1}^N [1 - e^{-\lambda_a T n}] |u[n]|^2 \sum_{n=1}^N [1 - e^{-\lambda_a T n}] |v[n]|^2} \quad (12)$$

for the random arrival of the PU. Note that  $T_d = T_a$  when  $\lambda_d = \lambda_a$ .

### IV. NUMERICAL RESULTS

In this section, we compare the spectrum sensing performance of the proposed scheme with that of the conventional scheme of [6] in terms of ROC and detection probability. We assume the following parameters:  $N = 100$ ,  $\lambda_d T = \lambda_a T = 1$ ,  $\alpha = 2f_c$ ,  $P_{fa} = 0.01, 0.03, 0.05, 0.15, 0.4$ , and 1, and a PU signal modulated by the binary phase shift keying with a carrier frequency  $f_c$  of 100 Hz. The thresholds are determined

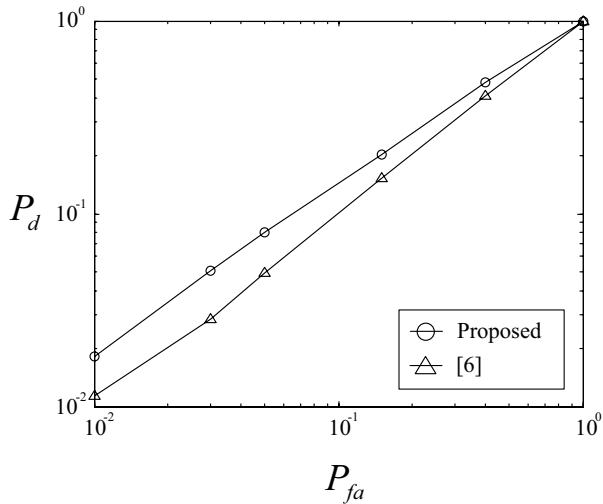


Fig. 2. ROC curves of the proposed and conventional schemes over AWGN channel in high traffic environments when SNR = -15 dB.

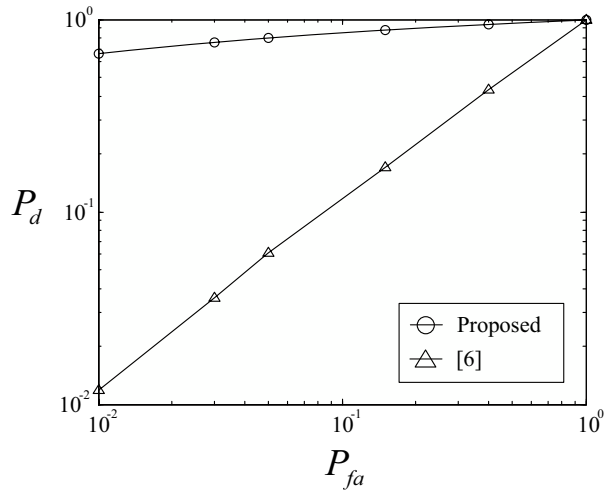


Fig. 4. ROC curves of the proposed and conventional schemes over AWGN channel in high traffic environments when SNR = -5 dB.

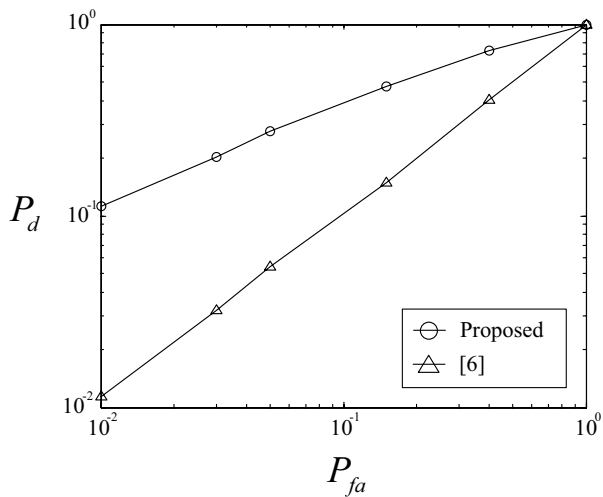


Fig. 3. ROC curves of the proposed and conventional schemes over AWGN channel in high traffic environments when SNR = -10 dB.

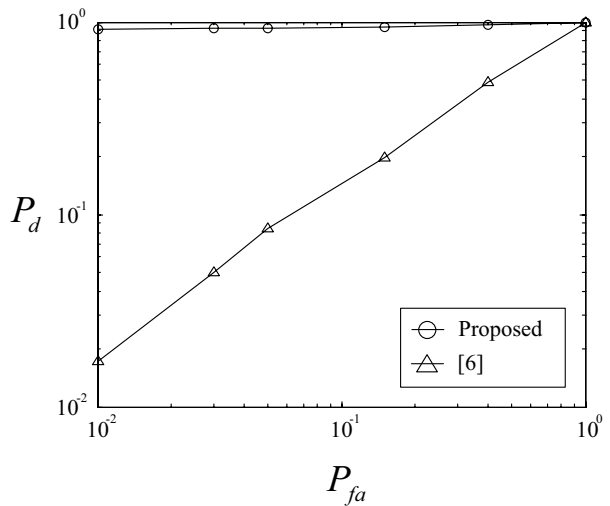


Fig. 5. ROC curves of the proposed and conventional schemes over AWGN channel in high traffic environments when SNR = 0 dB.

by choosing the test statistics satisfying the false alarm probabilities above in the absence of PU signals, thus, the thresholds for the proposed and conventional schemes would not have the same value but result in the same false alarm probability. It is also assumed that one of the random departure and arrival is chosen randomly with equal probability in each iteration of the simulation.

Figures 2-5 show the ROC curves of the proposed and conventional schemes over an AWGN channel in high traffic environments with the SNR values of -15 dB, -10 dB, -5 dB, and 0 dB, respectively, where  $P_d$  represents the detection probability defined as  $\Pr(H_1|H_1)$ . From the figures, it is clearly observed that the proposed scheme provides a significant performance improvement over the conventional scheme, and the improvement becomes more pronounced as the value of SNR increases. The conventional scheme achieves the worst case of the ROC performance at low SNRs such as -15 dB,

-10 dB, and -5 dB since the energy detection approach used in the conventional scheme can scarcely distinguish the signal from the noise in such low SNR environments, whereas the cyclostationarity of the signal is easily distinguishable regardless of the SNR value since the AWGN is not a cyclostationary process, and thus, the proposed scheme can generally provide a better ROC performance than the conventional scheme.

Figures 6-7 show the detection probabilities of the proposed and conventional schemes over AWGN channel in high traffic environments as a function of the SNR when false alarm probabilities are 0.01 and 0.15, respectively. From the figures, we can see that the proposed scheme has a better detection performance than the conventional scheme in both high and low SNRs.

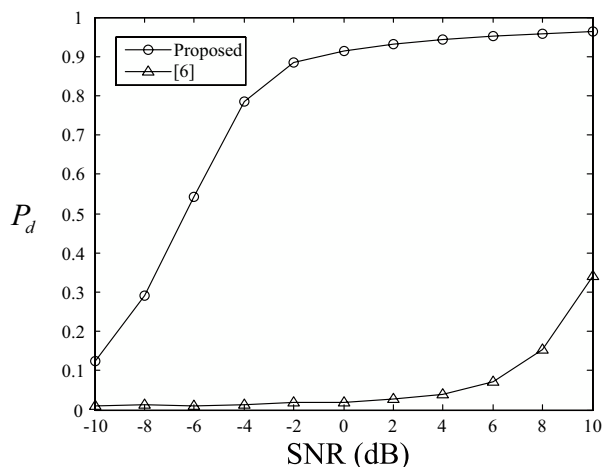


Fig. 6. The detection probabilities of the proposed and conventional schemes over AWGN channel in high traffic environments as a function of SNR when  $P_{fa} = 0.01$ .

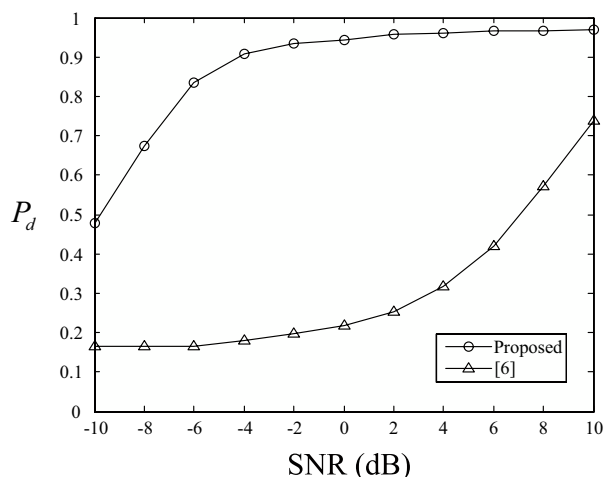


Fig. 7. The detection probabilities of the proposed and conventional schemes over AWGN channel in high traffic environments as a function of SNR when  $P_{fa} = 0.15$ .

## V. CONCLUSION

In this paper, we have proposed a spectrum sensing scheme based on the cyclostationarity of PU signals in high traffic environments. We have first modeled the spectrum sensing problem in high traffic environments as a binary hypothesis testing problem and developed the corresponding GLR. Applying an estimate of the squared magnitude of the SAF to the GLR, we have proposed a test statistic. The spectrum sensing performance of the proposed scheme has been compared with that of the conventional scheme. It has been confirmed that the proposed scheme provides a significant improvement over the conventional scheme in high traffic environments.

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