Prediction of Soil Hydraulic Conductivity from Particle-Size Distribution

A.F. Salarashayeri and M. Siosemarde

Abstract—Hydraulic conductivity is one parameter important for predicting the movement of water and contaminants dissolved in the water through the soil. The hydraulic conductivity is measured on soil samples in the lab and sometimes tests carried out in the field. The hydraulic conductivity has been related to soil particle diameter by a number of investigators. In this study, 25 set of soil samples with sand texture. The results show approximately success in predicting hydraulic conductivity from particle diameters data. The following relationship obtained from multiple linear regressions on data $(R^2 = 0.52)$:

$$K_s = 10.06 + 118.54(d_{10}) - 12.50(d_{50}) - 7.32(d_{60})$$

Where d_{10}, d_{50} and d_{60}, are the soil particle diameter (mm) that 10%, 50% and 60% of all soil particles are finer (smaller) by weight and K_s, saturated hydraulic conductivity is expressed in m/day. The results of regression analysis showed that d_{10} play a more significant role with respect to K_s, saturated hydraulic conductivity (m/day), and has been named as the effective parameter in K_s calculation.

Keywords—Hydraulic conductivity, particle diameter, particle-size distribution and soil

I. INTRODUCTION

SATURATED hydraulic conductivity represents the ability of a porous media to transmit water through its voids [2, 13, 15]. Since, direct measurement of hydraulic conductivity is time consuming and costly, indirect methods such as predicting from readily available soil properties e.g. particle-size distribution have been developed [2, 5, 16, 19 & 27]. Many different techniques have been proposed to determine estimate saturated hydraulic conductivity, including field methods, laboratory methods and calculations from empirical formulae [22]. Although in hydromechanics, it would be more useful to characterize the diameters of pores rather than those of the grains, the pore size distribution is very difficult to determine, so that approximation of hydraulic properties are mostly based on the easy-to-measure grain size distribution as a substitute [7]. There have been attempts to estimate saturated hydraulic conductivity based on particle-size distribution *(PSD)* [3, 16, 23, 25, 26, 27].Freeze and Cherry (1979) has long been recognized that hydraulic conductivity is related to the grain-size distribution of granular porous media [9]. Hazen (1982) proposed the following relationship between saturated hydraulic conductivity and soil particle diameter:

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$$K_S = c(d_{10})^2 \tag{1}$$

Where K_s is expressed in cm/sec, c is a constant that varies from 1.0 to 1.5, and d_{10} is the soil particle diameter (mm) such that 10% of all soil particles are finer (smaller) by weight [8 & 11]. Shepherd (1989) extended Hazen's research by performing power regression analysis [20].

Also Uma et al. (1989) suggested an equation to estimate the Ks and transmissivity of sandy aquifers of the same form as Hazen Equation [24]. Puckett et al. (1985) sampled six soils at seven different locations in the Alabama lower coastal plain [17], and used regression analysis to determine that percentage of clay sized particles was the best predictor of Ks. Rawls and Brakensiek (1989) used field data across the U.S. to develop a regression equation that relates porosity, and the percentages of sand and clay-sized particles in the sample to K_s [18]. Jabro (1992) estimated K_s from grain-size and bulk density data [12].

Ahuja et al. (1989) estimated Ks using the generalized form of the Kozeny-Carmen equation [1]. Alyamani and Sen (1993) proposed the relationship between saturated hydraulic conductivity and soil particle diameters for 32 sandy soil samples obtained in Saudi Arabia and Australia with the equation [2]:

$$K_s = 1.505[I_o + 0.025(d_{50} - d_{10})]^2 \tag{2}$$

Where K_s is expressed in cm/sec, I_o is the x-intercept of the straight line formed by joining d_{50} and d_{10} of the grain-size distribution curve (mm). d_{50} is the mean grain-size for which 50% of the particles are finer by weight (mm). Sperry and Peirce (1995) developed a linear model to estimate K_s based on grain size, shape, and porosity [21]. [14] sought to improve upon K_s prediction methods by quantifying the characteristics of the pore spaces at a microscopic scale. [8] developed multiple linear regression for southeastern U.S. sandy soils based on regional soil data.

[10] developed a new model to estimate saturated hydraulic conductivity from soil structural properties derived from water retention curve.

[5] reported that considerable success in predicting hydraulic conductivity from PSD data of soils. [13] reported that the lower content of both silt and organic matter and lower values of bulk density had increased K_s.

The results showed that the hydraulic conductivities calculated by the USBR and Slitcher methods are in all cases lower than for the other methods [6, 28 & 29]. Hazen formula which is based only on the d_{10} particle size is less accurate than the Kozeny-Carman formula which is based on the entire particle size distribution and particle shape [4 & 29].

The aim of the study was to determine relationship between saturated hydraulic conductivity and particle-size distribution.

II. MATERIALS AND METHODS

The 25 sets of soil samples were collected to estimate hydraulic conductivity based on particle-size distribution (PSD). Standard methods were applied to investigate particle size distribution (grain size curve), and finally determine of parameters of d_{10}, d_{50} and d_{60}, Where d_{10}, d_{50} and d_{60}, are the soil particle diameter (mm) that 10%, 50% and 60% of all soil particles are finer (smaller) by weight.

Soil texture was classified according to the International Society of Soil Science (ISSS) classification system. The soil texture was sand.

The values of parameters of d_{10}, d_{50}, d_{60} and saturated hydraulic conductivity are summarized in Table 1. The mean values of parameters of d_{10}, d_{50} and d_{60} were 0.253, 0.707 and 0.936 [mm], respectively, also the mean values of saturated hydraulic conductivity was 24.38 (m/day).

In this study saturated hydraulic conductivity was measured by the constant head method. The samples were first wetted by capillary for 24 hours. This was done from the bottom so that air could escape from the upper surface. The water is then allowed to flow through the soil with maintaining a constant pressure head and saturated hydraulic conductivity was measured when outflow rate becomes constant.

The results were analyzed with SPSS 16.0 and EXCEL software with statistics such as Correlation Coefficient (R), Root Mean Square Error (RMSE), Mean Bias Error (MBE),

TABLE I
SUMMARIZE OF STATISTICS OF D_{10}, D_{50}, D_{60} AND SATURATED HYDRAULIC CONDUCTIVITY

Statistics	PARAMETERS			
	d_{10}	d_{50}	d_{60}	K_s
Mean	0.253	0.707	0.936	24.38
Minimum	0.16	0.42	0.61	15.1
Maximum	0.36	1.10	1.38	36.1
Std. Deviation	0.061	0.185	0.248	5.96
Skewness	-0.171	0.179	0.287	0.204

d_{10}, d_{50} and d_{60}, are the soil particle diameter (mm) that 10%, 50% and 60% of all soil particles are finer (smaller) by weight and K_s, saturated hydraulic conductivity is expressed in m/day

Mean Absolute Error (MAE), and Relative Error (RE) that calculated using equation (3), (4), (5) and (6) respectively, where n represents the number of instances presented to the model and O_i and P_i represents measured and predicted, and O_{ave} and P_{ave} represents mean values of measured and predicted respectively.

$$R = \left[\frac{\sum_{i=1}^{n}(O_i - O_{ave})(P_i - P_{ave})}{\sqrt{\sum_{i=1}^{n}(O_i - O_{ave})^2(P_i - P_{ave})^2}} \right]^{1/2} \quad (3)$$

$$RMSE = \left[\frac{\sum_{i=1}^{n}(P_i - O_i)^2}{n} \right]^{1/2} \quad (4)$$

$$MAE = \sum_{i=1}^{n}\left[|P_i - O_i|/n \right] \quad (5)$$

$$RE = (MAE/O_{ave})100 \quad (6)$$

III. RESULT AND DISCUSSION

The following equations for Ks, saturated hydraulic conductivity (m/day), were obtained from multiple regressions on data.

$$K_S = 8.91 + 61.08(d_{10}) \quad (7)$$
$$K_S = 16.88 + 10.60(d_{50}) \quad (8)$$
$$K_S = 16.55 + 8.32(d_{60}) \quad (9)$$
$$K_S = 16.16 + 121.5(d_{10})^2 \quad (10)$$
$$K_S = 20.90 + 6.52(d_{50})^2 \quad (11)$$
$$K_S = 20.79 + 3.84(d_{60})^2 \quad (12)$$
$$K_S = 10.14 + 114.67(d_{10}) - 20.93(d_{50}) \quad (13)$$
$$K_S = 9.80 + 116.39(d_{10}) - 15.92(d_{60}) \quad (14)$$
$$K_S = 16.68 - 2.85(d_{50}) + 10.38(d_{60}) \quad (15)$$
$$K_S = 10.06 + 118.54(d_{10}) - 12.50(d_{50}) - 7.32(d_{60}) \quad (16)$$

Where d_{10}, d_{50} and d_{60}, are the soil particle diameter (mm) that 10%, 50% and 60% of all soil particles are finer (smaller) by weight and K_s, saturated hydraulic conductivity is expressed in m/day.

Table II was indicated the various statistics of equations mentioned above.

The results showed as per the table the equation (16) was the best model for predicting K_s, saturated hydraulic conductivity (m/day), with 0.719 R, 4.06 RMSE, 3.32 MAE and 13.62 RE. Comparison of observed vs. predicted values of saturated hydraulic conductivity obtained from the equation (16) as a 1:1 scale has been depicted in figure (1) that indicates good match.

TABLE II
SUMMARIZE OF STATISTICS OF VARIOUS EQUATIONS OF HYDRAULIC CONDUCTIVITY

Equation	STATISTICS PARAMETERS			
	R	RMSE	MAE	RE
7	0.621	4.58	3.68	15.08
8	0.329	5.52	4.58	18.79
9	0.346	5.48	4.51	18.49
10	0.617	4.60	3.74	15.33
11	0.295	5.58	4.65	19.08
12	0.311	5.55	4.56	18.72
13	0.715	4.09	3.37	13.82
14	0.712	4.10	3.26	13.39
15	0.346	5.48	4.50	18.44
16	0.719	4.06	3.32	13.62

R is the Correlation Coefficient; RMSE is the Root Mean Square Error; MAE, Mean Absolute Error and RE, is the Relative Error

Fig. 1 Comparison of measured saturated hydraulic conductivity, K_s (m/day) and K_s estimated by equation (16)
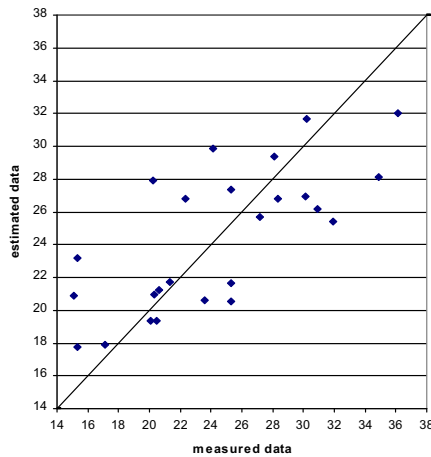

Fig. 2 Comparison of measured saturated hydraulic conductivity, K_s (m/day) and K_s estimated by equation (7)

The results showed that among single parameter linear equations (equation 7, 8 and 9) in this study, the equation that predicted K_s, saturated hydraulic conductivity (m/day), from d_{10} estimated better (less prediction error) than d_{50} and d_{60} with 0.621 R; 4.58 RMSE; 3.68 MAE; and 15.08 RE and the equation that predicted K_s, from d_{50} and d_{60} estimated with larger prediction error and the higher trend is evident between Ks and d_{10}. The results of single parameter regression analysis showed that when d_{10}, d_{50} and d_{60} increase, Ks, saturated hydraulic conductivity (m/day), increases. The results showed that among single parameter quadratic equations (equation 10, 11 and 12) in this study, the equation that predicted K_s, saturated hydraulic conductivity (m/day), from d_{10} estimated better (less prediction error) than d_{50} and d_{60} with 0.617 R; 4.60 RMSE; 3.74 MAE; and 15.33 RE. Comparison between linear and quadratic single parameter equations showed K_s, saturated hydraulic conductivity predicted from linear equations, estimated rarely better than quadratic single parameter equations.

Also the results showed that among tow parameter linear equations (equation 13, 14 and 15), the equation 13 that predicted K_s, from d_{10} and d_{50} (without d_{60}) estimated better than other tow parameter equations with 0.715 R; 4.09 RMSE; 3.37 MAE; and 13.82 RE and K_s predicted based on d_{50} and d_{60} (without d_{10}) estimated with largest prediction error. Then it is concluded that d_{10} play a more significant role with respect to K_s, saturated hydraulic conductivity (m/day), and has been named as the effective parameter in K_s calculation.

Variations between predicted and observed K_s are reported in the literature [2, 5, 12, 16, 18, 23, 24, 25, 26 & 27], and the results showed that when three parameter was used as input of linear equations for predicting K_s, estimated K_s better than single and tow parameter equations.

Also the Comparison between observed and predicted data obtained from the equation (7), (8), (9), (10), (11), (12), (13), (14) and (15) have been depicted (Fig. 2-10).
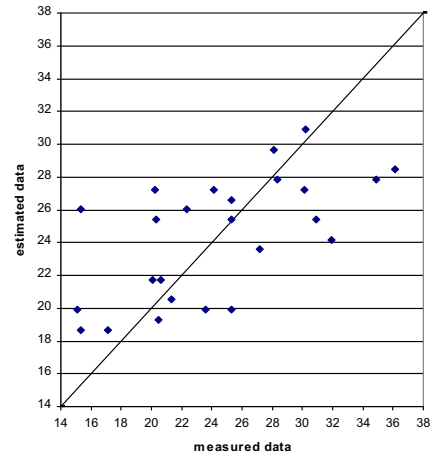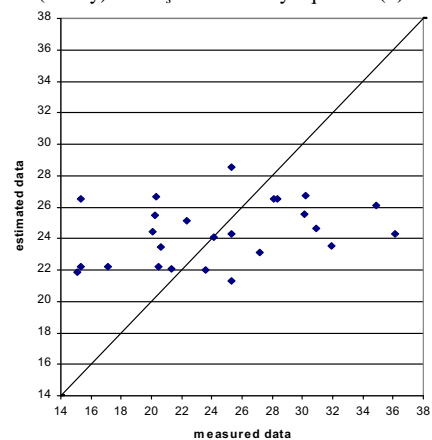

Fig. 3 Comparison of measured saturated hydraulic conductivity, K_s (m/day) and K_s estimated by equation (8)
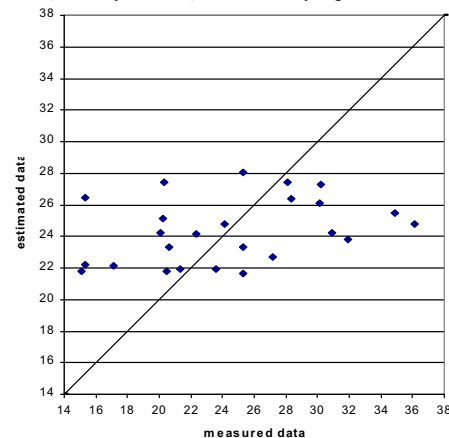

Fig. 4 Comparison of measured saturated hydraulic conductivity, K_s (m/day) and K_s estimated by equation (9)
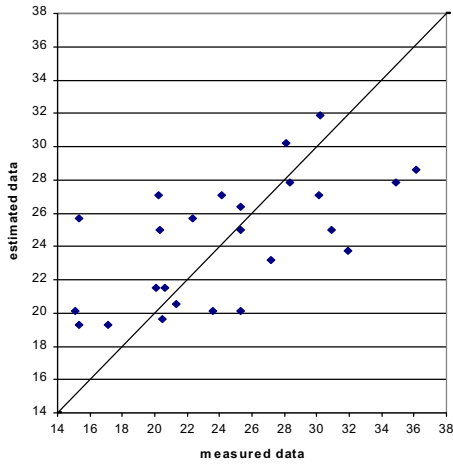
Fig. 5 Comparison of measured saturated hydraulic conductivity, K_s (m/day) and K_s estimated by equation (10)
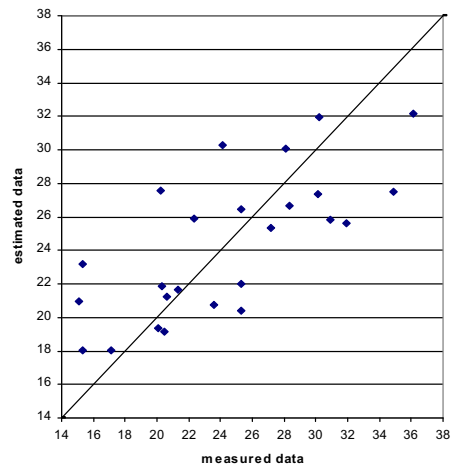

Fig. 8 Comparison of measured saturated hydraulic conductivity, K_s (m/day) and K_s estimated by equation (13)
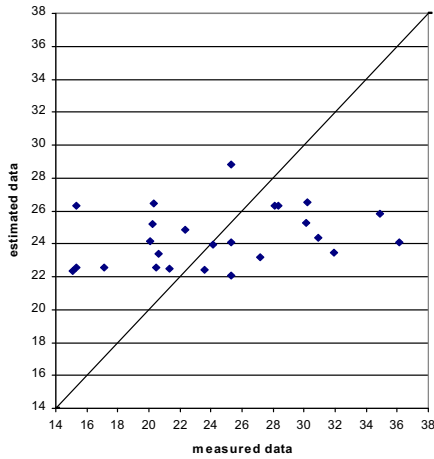

Fig. 6 Comparison of measured saturated hydraulic conductivity, K_s (m/day) and K_s estimated by equation (11)
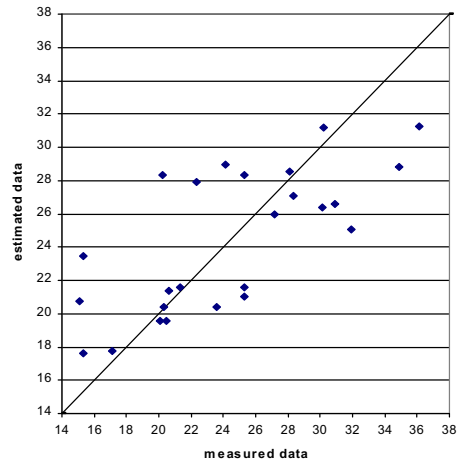

Fig. 9 Comparison of measured saturated hydraulic conductivity, K_s (m/day) and K_s estimated by equation (14)
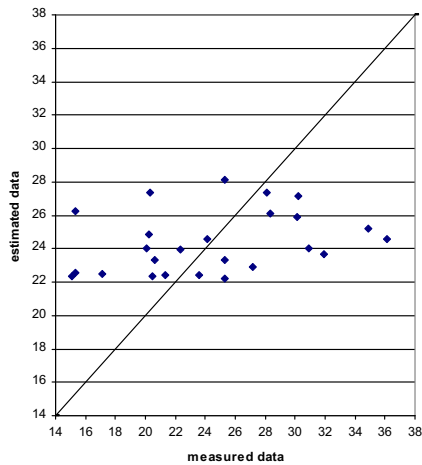

Fig. 7 Comparison of measured saturated hydraulic conductivity, K_s (m/day) and K_s estimated by equation (12)
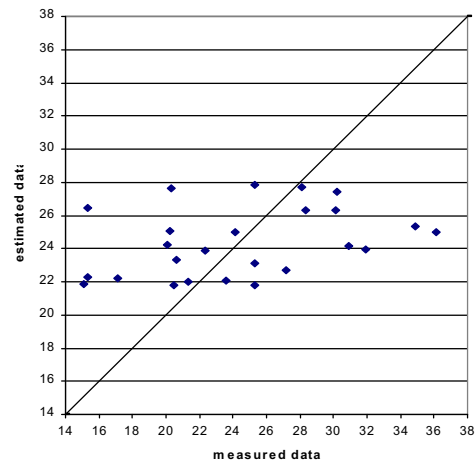

Fig. 10 Comparison of measured saturated hydraulic conductivity, K_s (m/day) and K_s estimated by equation (15)

IV. CONCLUSION

In this study described equations to estimate Ks, saturated hydraulic conductivity, from d_{10}, d_{50} and d_{60} data. The results showed approximately success in predicting hydraulic conductivity from particle diameters data. The results of regression analysis showed that d_{10} play a more significant role with respect to K_s, saturated hydraulic conductivity (m/day), and has been named as the effective parameter in K_s calculation. Comparison between linear and quadratic single parameter equations showed K_s, saturated hydraulic conductivity predicted from linear equations, estimated rarely better than quadratic single parameter equations.

REFERENCES

[1] Ahuja, L.R., D.K. Cassel, R.R. Bruce, and B.B. Barnes. 1989. Evaluation of spatial distribution of hydraulic conductivity using effective porosity data. Soil Science 148, no. 6: 404-411.
[2] Alyamani, M.S. and Z. Sen. 1993. Determination of hydraulic conductivity from complete grain-size distribution curves. Ground Water 31, no. 4: 551-555.
[3] Boadu, F. K. 2000. Hydraulic Conductivity of Soils from Grain-Size Distribution: New Models. Journal of Geotechnical and Geoenvironmental Engineering.
[4] Carrier, W.D. 2003. Goodbye, Hazen; Hello, Kozeny-Carman. Journal of Geotechnical and Geoenvironmental Engineering.1054.
[5] Chakraborty, D., A. Chakraborty, P. Santra, R. K. Tomar1, R. N. Garg, R. N. Sahoo1, S. G. Choudhury, M. Bhavanarayana and N. Kalra. 2006. Prediction of hydraulic conductivity of soils from particle-size distribution. Current Science, VOL. 90, NO. 11, 1527-1531.
[6] Cheng, C., and Chen, X. 2007. Evaluation of Methods for Determination of Hydraulic Properties in an Aquifer- Aquitard System Hydrologically Connected to River. Hydrogeology Journal. 15: 669-678
[7] Cirpka, O. A.2003. Environmental Fluid Mechanics I: Flow in Natural Hydrosystems.
[8] Cronican A. E. and M. M. Gribb. 2004. Literature review: Equations for predicting hydraulic conductivity based on grain-size data. Supplement to Technical Note entitled: Hydraulic conductivity prediction for sandy soils. Published in Ground Water, 42(3):459-464.
[9] Freeze, R. A., and Cherry, J. A. 1979. Groundwater. Prentice Hall Inc., Englewood Cliffs, New Jersey.
[10] Han, H., D. Gimenez, L. Lilly. 2008. Textural averages of saturated soil hydraulic conductivity predicted from water retention data. Geoderma, 146:121-128.
[11] Hazen, A. 1892. Some physical properties of sands and gravels. Massachusetts State Board of Health, Annual Report, 539-556.
[12] Jabro, J.D. 1992. Estimation of saturated hydraulic conductivity of soils from particle size distribution and bulk density data. Journal of the American Society of Agricultural Engineers 35, no. 2: 557-560.
[13] Jadczyszyn, J. and J. NiedŸwiecki. 2005. Relation of Saturated Hydraulic Conductivity to Soil Losses. Polish Journal of Environmental Studies Vol. 14, No 4, 431-435.
[14] Lebron, I., M.G. Schaap, and D.L. Suarez. 1999. Saturated hydraulic conductivity prediction from microscopic pore geometry measurements and neural network analysis. Water Resources Research 35, no. 10: 3149-3158.
[15] Mallants D., Jaques D., Tseng P. H., Van Genuchten M. Th., Feyen J. 1997. Comparison of three hydraulic property measurement methods. J. Hydrol. 199, 295.
[16] Mualem, Y., 1976. A new model for predicting the hydraulic conductivity of unsaturated porous media. Water Resour. Res., 12, 593–622.
[17] Puckett, W.E., J.H. Dane, and B.F. Hajek. 1985. Physical and mineralogical data to determine soil hydraulic properties. Soil Science Society of America Journal 49, no. 4:831-836.
[18] Rawls, W.J. and D.L. Brakensiek. 1989. Estimation of soil water retention and hydraulic properties. Unsaturated flow in Hydrologic Modeling Theory and Practice, ed. H. J.
[19] Shao, M. and Robert, H., 1998. Integral method of soil hydraulic properties. Soil Sci. Soc. Am. J., 62, 585–592.
[20] Shepherd, R.G. 1989. Correlations of Permeability and Grain Size. Ground Water 27, no. 5: 633-638.
[21] Sperry, J.M. and J.J. Peirce. 1995. A Model for Estimating the Hydraulic Conductivity of Granular Material Based on Grain Shape, Grain Size, and Porosity. Ground Water 33, no. 6: 892-898.
[22] Todd, D. K., and Mays, L.W. 2005.Groundwater Hydrology. John Wiley & Sons, New York.
[23] Tyler, S. W. and Wheatcraft, S. W., 1989. Application of fractal mathematics to soil water retention estimation. Soil Sci. Soc. Am. J., 53, 987–996.
[24] Uma, K.O., B.C.E. Egboka, and K.M. Onuoha. 1989. New statistical grain-size method for evaluating the hydraulic conductivity of sandy aquifers. Journal of Hydrology 108, 343-366.
[25] Van Dam J. C., Stricker, J. N. M. and Droogers, P., 1992. Inverse method for determining soil hydraulic function from one-step outflow experiments. Soil Sci. Soc. Am. J., 56, 1042–1050.
[26] Van Genuchten, M. Th., 1980. A closed form equation for predicting the hydraulic conductivity of unsaturated soils. Soil Sci. Soc. Am. J., 44, 892–898.
[27] Van Genuchten, M. Th. and Leji, F., 1989. On estimating the hydraulic properties of unsaturated soils. In Proceedings of the International Workshop on Indirect Method of Estimating Hydraulic Properties of Unsaturated Soils (eds van Genuchten, M. Th. et al.), 11–13 October, US Salinity Laboratory and Department of Soil and Environmental Science, Univ. of California, Riverside, 1992, pp. 1–14.
[28] Vukovic, M., and Soro, A. 1992. Determination of Hydraulic Conductivity of Porous Media from Grain-Size Composition. Water Resources Publications, Littleton, Colorado.
[29] Udong, J. 2007. Evaluation of Empirical Formulae for Determination of Hydraulic Conductivity based on Grain-Size Analysis. Journal of American Science, 3(3), 54-60.