

# Rheodynamic Lubrication of a Rectangular Squeeze Film Bearing with an Exponential Curvature Variation using Bingham Lubricants

K. P. Vishwanath and A. Kandasamy

**Abstract**—The present work deals with analyses of the effects of bearing curvature and non-Newtonian characteristics on the load capacity of an exponential rectangular squeeze film bearing using Bingham fluids as lubricants. Bingham fluids are characterized by an yield value and hence the formation of a “rigid” core in the region between the plates is justified. The flow is confined to the region between the core and the plates. The shape of the core has been identified through numerical means. Further, numerical solutions for the pressure distribution and load carrying capacity of the bearing for various values of Bingham number and curvature parameter have been obtained. The effects of bearing curvature and non-Newtonian characteristics of the lubricant on the bearing performances have been discussed.

**Keywords**—rheodynamic lubrication, yield stress, non-Newtonian fluid, Bingham fluid, exponential squeeze film.

## I. INTRODUCTION

Lubrication of modern machines has been a challenging and emerging area of interest for tribologists. Although the researchers of the past have laid out a foundation of the squeeze film bearings, modern researchers intend to use non-Newtonian fluids as lubricants. Few researchers have used yield-stress fluids such as Bingham, Casson and Herschel-Bulkley fluids as lubricants. Researchers like Covey and Stanmore [4], Gartling and Phan-Thien [5], Donovan and Tanner [7], Huang et al [6], Batra and Kandasamy [2] and such others have worked on the Bingham squeeze flow problem. Cameron [3] has analyzed the load capacity of a slider bearing with exponential wedge variation using a Newtonian lubricant but the effect of exponential squeeze films on the load capacity of the bearing using non-Newtonian fluids has not been addressed by any of the researchers.

IN the present work, the problem of a squeeze film bearing with converging and diverging squeeze films has been analysed. The effect of the boundary variation on the pressure distribution and load capacity of the bearing using Bingham lubricants has been investigated. During the operations of the bearings, the maximum viscous shearing stresses arise in the region of the plates. Therefore, there may be a region in

the film where the shearing stresses do not exceed the yield value of the lubricant and thereby a core with zero velocity gradient is formed. The flow occurs only in the region where the shear stress exceeds the yield value. The flow is confined to the region between the core and the plates of the bearing. The shape of the core has been identified through numerical means. Further, numerical solutions have been obtained for the bearing performances such as pressure distribution and load capacity for different values of Bingham number and for different shapes of the squeeze films. The properties of exponential squeeze films on the load capacity of the bearing are investigated through non-Newtonian effects of the fluids.

## II. MATHEMATICAL FORMULATION OF THE PROBLEM

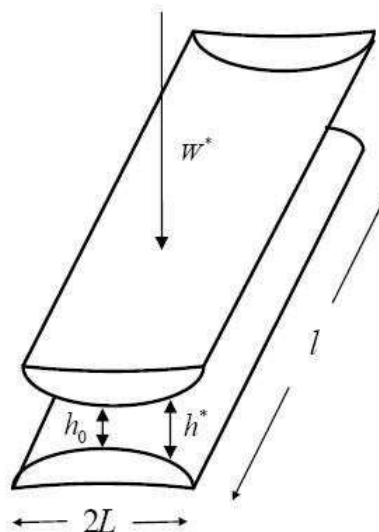


Fig. 1. Exponential Squeeze Film Bearing - Diverging type

The geometries of the problem are as shown in fig. 1 and fig. 2. We consider an isothermal, incompressible, steady flow of a time independent Bingham fluid squeezed between two sections of the bearing separated by a distance  $h^*$  such that  $h^* = h_0 e^{\frac{\alpha x}{L}}$  where  $h_0$  is the distance between the two sections at the mid point along the length of the bearing and  $\alpha$  is a constant which determines the curvature of the bearing. Let  $2L$  be the length of the bearing approaching each other with

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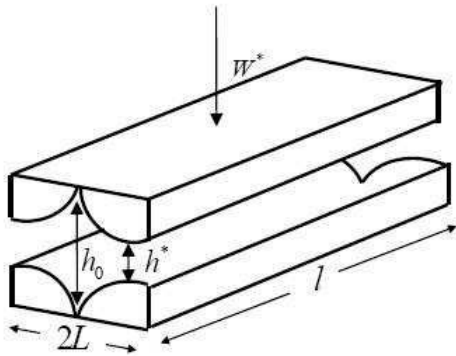


Fig. 2. Exponential Squeeze Film Bearing - Converging type

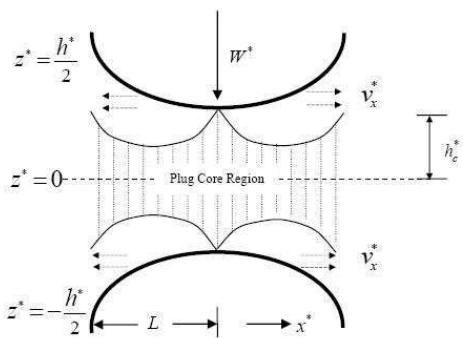


Fig. 3. Shape of the Core - Diverging type

a squeeze velocity  $v_s^*$  under a normal load  $W^*$ . We consider cartesian co-ordinates system  $(x^*, y^*, z^*)$  such that  $v_x^*$  and  $v_z^*$  represents the components of velocity in the direction of length and axis respectively and  $\rho$  denotes the density of the fluid. It is assumed that there is no sliding motion between the two plates.

The constitutive equation of a Bingham fluid is given by,

$$\tau_{ij} = 2 \left( \eta_1 + \frac{\eta_2}{I^{1/2}} \right) e_{ij}, \quad \left( \frac{1}{2} \tau_{ij} \tau_{ij} \geq \eta_2^2 \right) \quad (1)$$

where  $\tau_{ij}$  are the deviatoric stress components,  $\eta_1$  and  $\eta_2$  are constants, viz., plastic viscosity and yield value respectively,  $e_{ij}$  represents the rate of deformation components and  $I = 2e_{ij}e_{ij}$  is strain invariant. Practical examples of such materials are lubricating greases, colloidal suspensions, starch pastes, slurries, silicon suspensions and such other fluids.

Applying the basic assumptions of lubrication theory for thin films, the governing equations for the above squeeze film system in the non-dimensional form is given by,

$$\frac{\partial p}{\partial x} = \frac{\partial \tau_{xz}}{\partial z} \quad (2)$$

$$\frac{\partial p}{\partial z} = 0 \quad (3)$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0 \quad (4)$$

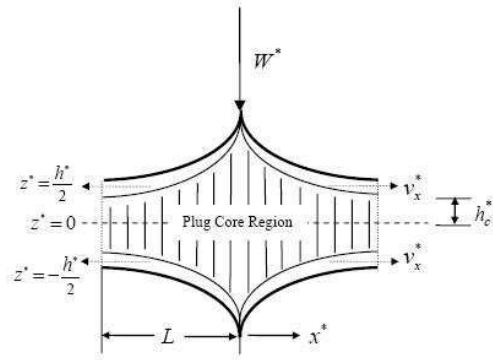


Fig. 4. Shape of the Core - Converging type

$$\tau_{xz} = B + \left| \frac{\partial v_x}{\partial z} \right| \quad (5)$$

where  $B = \frac{\eta_2 h_0^2}{L v_s \eta_1}$  is called the Bingham number.

The above equations (2), (3) and (5) together with the equation of continuity (4) are to be solved using the non-dimensional boundary conditions:

$$\begin{aligned} v_x &= 0 & \text{at } z &= -\frac{h}{2}, \frac{h}{2} \\ \frac{\partial v_x}{\partial z} &= 0 & \text{at } z &= 0 \\ v_z &= 0 & \text{at } z &= -\frac{h}{2}, \frac{h}{2} \\ v_z &= -1 & \text{at } z &= \frac{h}{2} \\ p &= p_a & \text{at } x &= 1 \end{aligned} \quad (6)$$

and  $v_x$  and  $\frac{\partial v_x}{\partial z}$  are continuous at the boundaries of the core,  $z = \pm h_c(x)$ . Here  $p_a$  is the non-dimensional atmospheric pressure,  $h = \frac{h^*}{h_0}$  and  $x = \frac{x^*}{L}$ .

### III. SOLUTION TO THE PROBLEM

The integral form of the continuity equation, also called the equation of squeeze motion, is given by

$$\int_0^{\frac{h}{2}} v_x dz = \frac{x}{2} \quad (7)$$

Now, equation (3) infers that pressure is independent of  $z$  and hence equation (2) can be integrated as,

$$\tau_{xz} = \frac{dp}{dx} z + c_1 \quad (8)$$

Substituting for  $\tau_{xz}$  from equation (8) into equation (5) and integrating the resulting equation using the boundary conditions (6), we get the velocity distribution in the upper half of the schematic representation as follows:

$$v_x = \left( \frac{dp}{dx} \right) \left[ \frac{(z - h_c)^2}{2} - \frac{(h - 2h_c)^2}{8} \right] \text{ in } h_c(x) \leq z \leq \frac{h}{2} \quad (9)$$

and the velocity in the plug core region can be written as,

$$v_c = \left( \frac{dp}{dx} \right) \left[ -\frac{(h - 2h_c)^2}{8} \right] \text{ in } 0 \leq z \leq h_c(x) \quad (10)$$

Considering the equilibrium of an element of the core in the fluid, we get,

$$\frac{dp}{dx} = -\frac{B}{h_c} \quad (11)$$

where  $2h_c$  is the thickness of the rigid core in the squeeze flow region.

Evaluating the equation of squeeze motion (7) by substituting equations (9) and (10), we get the pressure gradient in the axial direction as given below:

$$\frac{dp}{dx} = -\frac{12x}{4h_c^3 - 3e^{2\alpha x}h_c + e^{3\alpha x}} \quad (12)$$

Eliminating  $\frac{dp}{dx}$  from equations (11) and (12), re-arranging the resulting equality and substituting for  $h$ , we get,

$$h_c^3 - 3\left(\frac{e^{2\alpha x}}{4} + \frac{x}{B}\right)h_c + \frac{e^{3\alpha x}}{4} = 0 \quad (13)$$

Equation (13) is called the equation of core thickness where  $h_c = h_c(\alpha, x, B)$  and can be evaluated using any numerical iterative technique.

Further, integrating equation (12) between the limits  $x$  and 1 by substituting the numerical values of  $h_c$  obtained from equation (13), we get

$$p - p_a = \int_x^1 \frac{12x}{4h_c^3 - 3e^{2\alpha x}h_c + e^{3\alpha x}} dx \quad (14)$$

where equation (14) can be integrated using any quadrature formula and hence the pressure distribution along the axial direction can be obtained.

Again, the non-dimensionalised load carrying capacity of the bearing can be obtained by the integral formula,

$$w = \int_0^1 (p - p_a) x dx \quad (15)$$

#### IV. RESULTS AND DISCUSSIONS

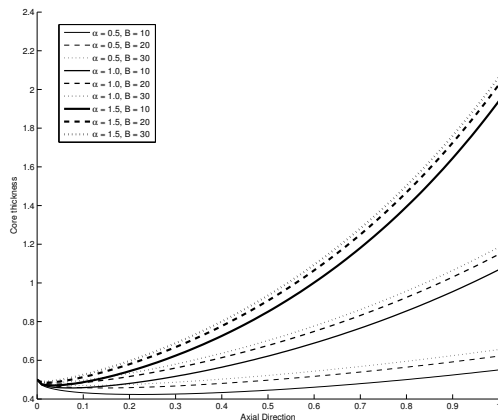


Fig. 5. Core Thickness in the axial direction when  $\alpha > 0$

The behavior of core thickness  $h_c$  for various values of  $\alpha$  and Bingham number  $B$  along the axial direction  $x$  is

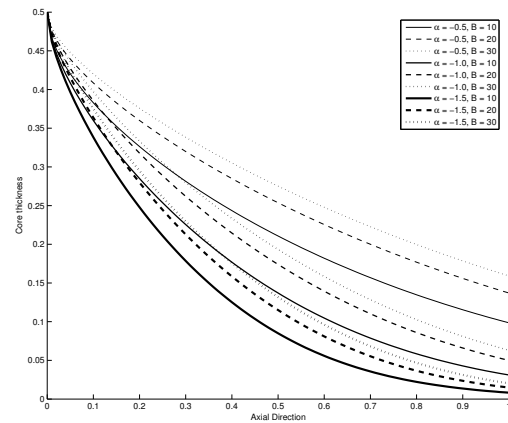


Fig. 6. Core Thickness in the axial direction when  $\alpha < 0$

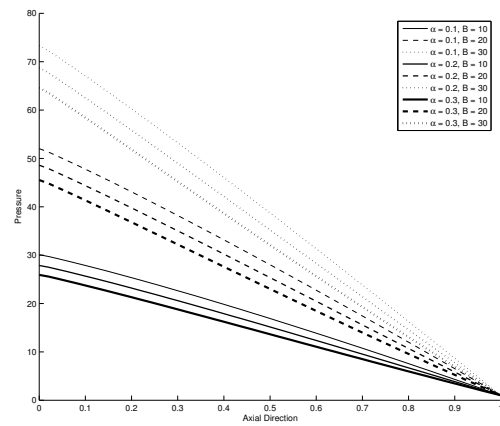
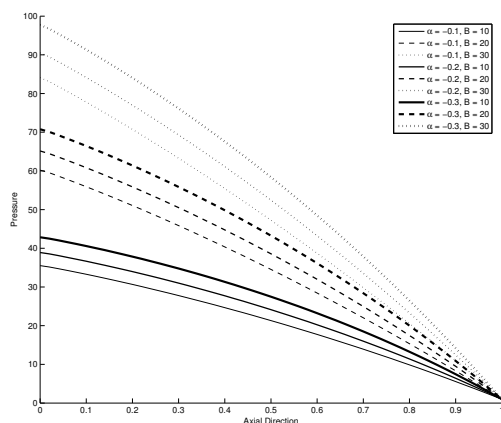
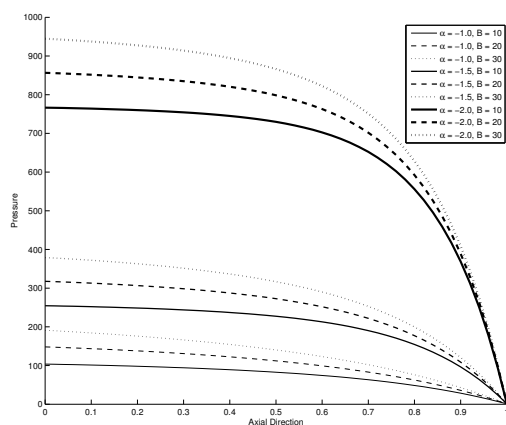


Fig. 7. Pressure Distribution in the axial direction when  $\alpha > 0$

computed and the results are depicted in figs. 5-6. When  $\alpha$  is positive, the core thickness is maximum at the periphery and minimum at the center. The thickness and shape of the core is found to increase along the axial direction for a particular Bingham fluid.  $h_c$  is found to increase with the increase in Bingham number but the rate of increase in  $h_c$  with respect to the Bingham number decreases with increase in curvature parameter  $\alpha$ . Again,  $h_c$  increases with increase in  $\alpha$  for a particular Bingham fluid. When  $\alpha < 0$  is negative, the core thickness is maximum at the center and minimum at the periphery. The thickness and shape of the core is found to decrease along the axial direction for a particular Bingham fluid. Again,  $h_c$  is found to increase with the increase in Bingham number and the rate of increase in the core thickness with respect to the Bingham number decreases with the decrease in curvature parameter  $\alpha$ . Again,  $h_c$  decreases with the decrease in  $\alpha$  for a particular Bingham fluid.

The pressure distribution along the axial direction has been computed for different values of the curvature parameter and

Fig. 8. Pressure Distribution in the axial direction when  $-0.1 \leq \alpha \leq -0.3$ Fig. 9. Pressure Distribution in the axial direction when  $\alpha \leq -1.0$ 

Bingham number and the results are as shown in figs. 7 - 9. The pressure distribution decreases from a maximum to zero along the axial direction. When  $\alpha$  is greater than zero, it can be observed that pressure decreases with the increase of curvature parameter  $\alpha$  for a constant Bingham number and the rate of decrease of pressure with respect to the Bingham number is found to be significant only for lower values of  $\alpha$  as shown in fig. 7. When  $\alpha$  is less than zero, it can be observed that pressure increases with the decrease of curvature parameter  $\alpha$  for a constant Bingham number. Further, pressure increases with the increase in Bingham number for both types of squeeze film bearings as shown in figs. 7 and 8.

The results of load carrying capacity for various values of curvature parameter  $\alpha$  and Bingham number are tabulated in tables I and II. From these results, we observe that the load capacity of the bearing decreases with the increase in curvature for a particular Bingham fluid. Further, it can be seen that fluids with high Bingham number infer a squeeze film bearing with high load carrying capacity.

TABLE I  
LOAD CAPACITY VALUES FOR BEARINGS OF CONVERGING TYPE

$\alpha \rightarrow$ B $\downarrow$	-1.5	-1.0	-0.5
10	84.0527	29.2497	12.4774
20	99.3984	39.1317	18.7715
30	114.3700	48.6461	24.8033

TABLE II  
LOAD CAPACITY VALUES FOR BEARINGS OF DIVERGING TYPE

$\alpha \rightarrow$ B $\downarrow$	0.5	1.0	1.5
10	3.8866	2.5925	1.8673
20	6.5285	4.3657	3.0852
30	9.0495	6.1008	4.2834

To the best of our knowledge, no work is available for exponential squeeze film bearing with time-independent non-Newtonian fluids with yield stress. The computed results of the load capacity corresponding to  $\alpha = 0$  are found to be in agreement with the results obtained by Kandasamy [2] (corresponding to the case of Reynolds number,  $Re = 0$ ) and Vishwanath and Kandasamy [8] (corresponding to the case of Reynolds number,  $Re = 0$  and power law index,  $n = 1.0$ ).

## V. CONCLUSION

From the investigation, we can conclude that the curvature effects of the bearing contribute significantly to the load capacity of the bearing and fluids with high Bingham number and very low value of curvature parameter  $\alpha$  ( $\alpha < 0$ ) will have a high bearing load capacity.

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