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A Dynamic Hybrid Option Pricing Model by Genetic Algorithm and Black- Scholes Model

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***Abstract*—**Unlike this study focused extensively on trading behavior of option market, those researches were just taken their attention to model-driven option pricing. For example, Black-Scholes (B-S) model is one of the most famous option pricing models. However, the arguments of B-S model are previously mentioned by some pricing models reviewing. This paper following suggests the importance of the dynamic character for option pricing, which is also the reason why using the genetic algorithm (GA). Because of its natural selection and species evolution, this study proposed a hybrid model, the Genetic-BS model which combining GA and B-S to estimate the price more accurate. As for the final experiments, the result shows that the output estimated price with lower MAE value than the calculated price by either B-S model or its enhanced one, Gram-Charlier garch (G-C garch) model. Finally, this work would conclude that the Genetic-BS pricing model is exactly practical.

***Keywords*—**genetic algorithm, Genetic-BS, option pricing model.

I. INTRODUCTION

RAPID changes in the market often let previous studies to focus on the theory development but not be applied them into the real market. That is the fact: the financial investment, an application, has become more complex than it as usual. The real dilemma is that the market price and the theoretical price are inconsistent at the same time so that investors are not able easily to predict the prices accurately. Moreover, traditional option pricing models have not been brought to public attention to evaluate option price, especially during the doubtful international economics. Therefore, in this study, the genetic algorithm, a dynamic evolution technique, was tried to be utilized to enhance the Black-Scholes model and obtained the real market volatility at the same.

II. LITERATURE

In this section, some researches put forward relative options evaluation model. For example, one famous option pricing model, the Black-Scholes model, and recent one, the Gram-Charlier garch model, respectively are described as below:

A. Black-Scholes model (B-S)

"The stock price follows a random walk in continuous time with a variance rate proportional to the square of the stock price" (Black & Scholes 1973) [1]. That rule was just mentioned in the Black-Scholes (B-S) model, a celebrated option pricing model. In the B-S model, such as the following formula expressions, stock index (S), exercise price (K), time until expiration (T), risk-free rate (r), volatility rate (σ), and the standard deviation of the returns for the underlying asset (Normal(.)) were taken into consideration in the model.

$$C = SN(d_1) - Ke^{-rT}N(d_2) \tag{1}$$

$$P = Ke^{-rT}N(-d_2) - SN(-d_1) \tag{2}$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{3}$$

$$d_2 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{4}$$

As for the equation (1)-(4), *C* is the calculated value to the call option price. *S* is stocks index. *K* is the exercise price. *P* is the calculated value to the put option price. *r* is the risk-free rate. *T* is the time to expiration. σ is the volatility of the stock price, which is the annualized standard deviation. *ln* is the natural logarithms. Thus, $N(d_i)$ is the cumulative distribution value for a standard normal random variable with value $d_{i.}$

However, all the detailed factors are enforced to the prior suppositions by the B-S theory. Such as: the standard deviation of the returns for the underlying asset looked up normal distribution table. By considering the specific argument, many studies have already tried to revise the controversy, the standard deviation of the B-S model. [2]– [6]

B. Gram-Charlier garch model (G-C garch)

As for the other pricing models, for instance, garch model, which is a time series model, owns two characteristics, Leptokurtosis and Fat-tail. However, Leptokurtosis and Fat-tail, are similar to the vibration characteristic of the real market. Thus, the garch model has proposed that the option price implied volatility smile curve is taken into consideration to the real market and then let option pricing easily [10].

Furthermore, owing to the doubtful distribution mentioned

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[2], Chou [7] employed the Gram-Charlier garch model (G-C garch). The model is also a hybrid approach combined the Gram-Charlier series and B-S model. Moreover, not only G-C garch model updates some parameters of B-S model but also considers the distribution of return on assets. The enhanced pricing expressions were as follows:

$$C_t = S_t e^{\delta\sigma_{\rho_T}} N(\tilde{d}) - Ke^{-rT} N(\tilde{d} - \sigma_{\rho_T}) + k_3 A_3 + (k_4 - 3)A_4 \quad (5)$$

$$P_t = C_t - S_0 + Ke^{-r_t T} \quad (6)$$

where ,

$$A_3 = \frac{1}{3!} S_t e^{\delta\sigma_{\rho_T}} \sigma_{\rho_T} \left\lfloor (2\sigma_{\rho_T} - \tilde{d}) n(\tilde{d}) - \sigma^2_{\rho_T} N(\tilde{d}) \right\rfloor \quad (7)$$

$$A_4 = \frac{1}{4!} S_t e^{\delta\sigma_{\rho_T}} \sigma_{\rho_T} \left[(\tilde{d}^2 - 1 - 3\sigma_{\rho_T}) n(\tilde{d}) + \sigma^3_{\rho_T} N(\tilde{d}) \right] \quad (8)$$

$$\tilde{d} = d + \delta \quad (9)$$

$$d = \frac{\ln\left(\frac{s_t}{k}\right) + \left(r_t T + \frac{1}{2}\sigma^2_{\rho\tau}\right)}{\sigma^2_{\rho\tau}} \quad (10)$$

$$\rho_T = \ln\left(\frac{S_T}{S_t}\right) \quad (11)$$

$$\delta = \frac{\mu_{\rho_T} - rT + \frac{\sigma^2_{\rho_T}}{2}}{\sigma_{\rho_T}} \quad (12)$$

In the G-C garch model, *S* is stock index. *n*(.) is the standard normal distribution probability function. *N*(.) is the standard normal distribution, the cumulative probability function. Average rate of return (σ_{ρ_T}) and standard deviation (μ_{ρ_T}) are under the risk neutral measure (*Q*). k_3 is an asset and the return of the kurtosis coefficient, and k_4 is the return on assets of the skewness coefficients. Next, the equation (11) is to describe the rate of return on assets. Owing to the G-C garch model composing of B-S model and Gram-Charlier series, European call option would be enhanced to equation (5), and the other, equation (6), the expression of put option is still a closed-form solution.

Furthermore, no matter how the accurately price that theoretical B-S model or its enhanced model, G-C garch one estimated, the estimated option price was not approximate to the real market price, in fact. However, the environment is dynamical and the market information is also changeable continuously. If the model could be developed based on the characteristic, the "change", the output price of this model might be practicable and close to the market price. Thus, the study would address a dynamic pricing model that based on the intensive concept. Consequently, the following statement would be the description of the proposed model.

III. RESEARCH PROCESS

Generally, the numerical option pricing models are classified into the model-driven approach and the data-driven approach [11]. About the model-driven approaches certainly mentioned by the prior description, people would always fall into the predicament because that the spot volatility and the rational environment parameters are unobservable and not easily obtained. Consequently, these kind models could still not be dynamically adjusted according to the real environment. For the development of the automatically adjusted ability, this research by the data-driven approach chooses a kind of artificial intelligence technique, the genetic algorithm, and tries to use it to propose a hybrid model to the commodity, the call/put option. That is, this kind hybrid model is capable of tuning dynamically to the possible environmental variables.

As for the B-S option pricing model's parameters, they were defined by many assumptions. Because of those assumptions, B-S model just became theoretical pricing model but not realized. Thus, the investors couldn't get option prices and didn't know how to trade in the real world even after they have surveyed the B-S model. Obviously, it is indeed the reason that the study proposed a dynamic option pricing model to argue those assumptions. Using dynamic option pricing to solve option pricing problems would be hopefully effective in this research.

In the research process detailed in Fig. 1, there are four stages respectively: the environmental variables collecting, the data pre-processing, the algorithm modeling, and the simulation finally. The most kernel stages is the third part which are adopted three models, the B-S model, the G-C garch model, and the proposed hybrid model (Genetic-BS model). About the prior two models, they has already discussed in the prior section. As for the proposed model, it would be described as the next section.

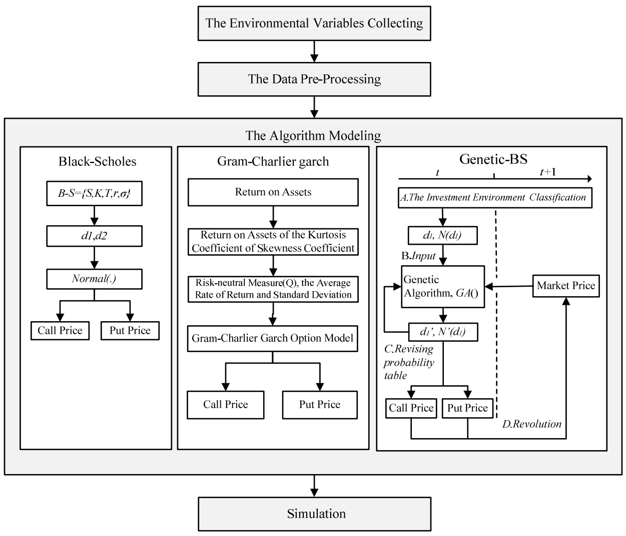

Fig. 1 Research Process

IV. THE PROPOSED G-BS MODEL

In the first stage, the assumption of the probability table mentioned in the B-S model, which is "the normal distribution", this research has different opinion. Considering the various investment ambiances, such as for long, for short or

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hedge and etc., each ambiance often maps to certain a specific probability table. For this, this research just tends to separate the entire investing period (environment) into various investment ambiances with different trends (attributes). That is, not all the option pricing is suited by the normal distribution. On the other hand, the variables of the B-S model deduced to those factors which embrace stocks index, exercise price, risk-free rate, the time until option expiration, the volatility, and hedge ratio are still indeed important to be referenced. Furthermore, a literature also pointed out the effectiveness approach which is the combination of genetic programming and the B-S model [8]. Owing to the application of dynamical adjusted method, the research tries to process a hybrid option pricing model which combines the genetic algorithm [6] and B-S model [3]. However, the following proposed model is different from the adaptive approach [8]. This model advanced to apply the GA to focus on the computing the rational probability table, which result is exactly not equal to the normal distribution. In figure 1, the proposed model we called (Genetic & B-S) model roughly including four steps:

a. Investment ambiances classification
b. Parameter normalization and encoding
c. Revising probability table
d. Revolution step

These steps are described next:

A. The Investment Ambiances Classification

First, the original collected option data contains call and put data. Moreover, as the prior description, *e*, is set to an attribute of the four kind trends, the set represented the four ambiances shown as follows:

$$e=\{"\vee", "\wedge", "/", "\backslash"\}; \quad (9)$$

where

"$\vee$" and "$\wedge$": the patterns denote the trend shown head and shoulders.

"/": the pattern denotes the trend shown up-trend.

"\": the pattern denotes the trend shown down-trend.

Besides, for observing the "down" trend and "up" trend conveniently, the entire dataset is advanced to be divide into two parts, the periods before and after the USA Sub-prime Mortgages. That means, the above simulation design can be formed to 16 (4*2*2) probability distribution tables.

B. Parameter Normalization and Encoding

In the second step, the parameters, d_1 and $N(d_1)$ of the Black-Scholes model are calculated from the collected option data, and next, the two parameters are immediately normalized and encoded. The range of d_1 and $N(d_1)$ is exactly $[U_{max}, U_{min}]=[0,1]$. Continuously, the two parameters are encoded in to a binary string. Encoding formula is defined as follows:

$$\delta = \frac{U_{max} - U_{min}}{2^{l} - 1} \quad (10)$$

For the equation (10), U_{max} is the upper bond value, and U_{min} is the lower bond.

To be similar, the output of the model is still represented as a binary string, which needs to be decoded. Equation (10) shows the translation equation, and then *x* is obtained. *x* means *N'(d1)*.

$$\chi = U_{min} + \left(\sum_{t=1}^{l} bi * 2^{i-1}\right) * \frac{U_{max} - U_{min}}{2^{l} - 1} \quad (11)$$

As for the *x*, more explain is described in the following paragraph.

C. Revising probability table

After the operation *B*, the new calculated value *x* is used to revise the probability table for each iteration. A adjusted rule is listed as equation (12), where *GA(e)* denotes as *x*. Finally, the newer version of the probability table is obtained. That is, we reform one normal distribution tables, denotes as *Normal(.)* to 16 probability distribution tables, denotes as *Normal'(.).*

$$Normal'(.) = \frac{GA(e) + Normal'(.)}{2}, \ whereGA(e) = x \quad (12)$$

Note that: each kind probability distribution table is mapping a kind of investment ambiance.

D. Revolution step

From step *A* to *C*, the three sequential steps complete the adjusting mechanism to revise the normal distribution table. Unfortunately, one adjusting process, an evolution, usually never been finished to be satisfied the terminate benchmark so that a next revolution after test the benchmark. As for the benchmark evaluation rule is listed as equation (13) whereas calculating the disparity value is between the normal distribution from of B-S model and the probability distribution from the Genetic-BS model.

$$Error = \left(\sum_{i=1}^{N} \left|\frac{\sigma_{GA} - \sigma_{Real}}{\sigma_{Real}}\right|\right) * \frac{1}{N} \quad (13)$$

Note that: the next revolution often starts at the time *t*+1, that is, after the steps *A* to *C*.

V. Simulation

In Fig.1, the final stage is the simulation of the three models and their comparison. The experiments result shows as following:

A. Data

The experimental data sources were Taiwan Index Options (TXO of Taiwan Futures Exchange), reported from the Taiwan Economy database (TEJ). The chosen daily data were from February 15, 2006 to November 5, 2008. Moreover, for the simulation comparison, the daily data were separated by U.S. Sub-prime Mortgages (June 27, 2007) into two parts, section A and section B. Thus, the section A is the period after Sub-prime

Mortgages, and the other period is section B. Data pre-processing is following the three steps ：

1) Filtering Data: Maturity of less than 22 days in recent months and less more than 500 option day trading volume are experimental data.
2) Missing Values: Delete the “no information”, ”null fields”, “blank field”. The process carried out is to avoid the errors in the program and is also able to reduce the chance of a miscarriage of justice.
3) Removing 10% of the Extreme Value: First, standardize the fluctuating rate, and remove the extreme top and the extreme last 10 % data.

B. Simulation

As mentioned above, the experimental results were clearly demonstrated that in different investment environment. Finally, using different probability distribution table to estimate the rational option price could increase forecasting ability.

Utilize the prior data such the data from the t time to train/obtain the “t+1” price. Following, test the time “t+1” estimated price and the time “t+1” spot price by B-S model, G-C garch model and Genetic-BS model respectively and then evaluate next.

C. Evaluate the performance

For the performance of evaluation, the average error of absolute values (MAE) calculated is used to evaluate the pricing accurate. MAE would comment B-S model, G-C garch model and G-BS model, which results showed as Table I.

Table Ⅰ showed that the option price obtained by using the Genetic-BS model is more realized value. This verifies that the proposed dynamic option pricing model is applicable. The more maximum MAE of G-C garch model is surprising to obtain. In this simulation, it seems that the worse outcome is indeed output by the G-C garch model.

TABLE I
MAE ESTIMATE COMPARISONS

Option Model	section B			section A		
	Call	**Put**	**Mean**	**Call**	**Put**	**Mean**
B-S model	2.47	0.99	1.73	3.13	1.00	2.06
G-C garch	3.90	1677.74	840.82	649.66	395.38	522.52
Genetic-BS model	0.56	0.71	0.63	0.67	0.72	0.70

Section B is the period before Sub-prime Mortgages, and the other period is section A. B-S model denotes as Black-Scholes model. Second row: G-C garch is Gram-Charlier garch model. Finally, Genetic-BS model is the approach presented the dynamic option pricing model. Then, call means Europe call option. Put represents put Europe option. Finally, Mean is the average value from call and put.

Obviously, this simulation result indicates that Genetic-BS model can estimate better and more exact accurate than B-S model even the G-C garch model.

VI. Conclusions

This study has enhanced Black-Scholes model by applying the genetic algorithm to solve option pricing problem dynamically. Especially, after the train step (revolution), using Genetic-BS model to price the option is still efficiency. In addition, for the complex financial environment, the Genetic-BS model is also more flexible than the others. All the above discussion is indeed the truly result.

Furthermore, it is also found that the experimental results are the same in Chou's research [7]. The G-C garch model was not suitable in the following two conditions: first, the duration is less than 21 days; second, the option volatility was higher than mediocre.

Overall, the Genetic-BS model we proposed observes that the dynamic option pricing models is capable to be improved the Black–Scholes model not only for one investing ambiance but others which is the major contribution of this research.

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