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Peak-to-Average Power Ratio Reduction in OFDM Systems using Huffman Coding

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Abstract— In this paper we proposed the use of Huffman coding to reduce the PAR of an OFDM system as a distortion-less scrambling technique, and we utilize the amount saved in the total bit rate by the Huffman coding to send the encoding table for accurate decoding at the receiver without reducing the effective throughput. We found that the use of Huffman coding reduces the PAR by about 6 dB. Also we have investigated the effect of PAR reduction due to Huffman coding through testing the spectral spreading and the inband distortion due to HPA with different IBO values. We found a complete match of our expectation from the proposed solution with the obtained simulation results.

***Index Terms*— HPA, Huffman coding, OFDM, PAR**

I. INTRODUCTION

Orthogonal frequency division multiplexing is a very attractive technique for high speed data transmission in mobile communications due to various advantages such as high spectral efficiency, robustness to channel fading, immunity to impulse interference, and capability of handling very strong multi-path fading and frequency selective fading without having to provide powerful channel equalization [1]. In recent years, several industrial standards based on OFDM have been emerged, such as the Terrestrial Digital Video Broadcast (DVB-T), the IEEE 802.11 Wireless Local Area Network (WLAN) scheme, as well as the IEEE 802.16 Broadband Wireless Access (BWA), particularly, Wireless Metropolitan Area Networks (IEEE 802.16d) WiMAX [2]. Besides a lot of advantages, some drawbacks become apparent, when using OFDM in transmission systems. A major obstacle is that the multiplex signal exhibits a very high peak-to-average power ratio (PAR). Therefore, nonlinearities may get overloaded by high signal peaks, causing inter-modulation among subcarriers and, more critical, undesired out-of-band radiation. If RF power amplifiers are operated without large power back-offs, it is impossible to keep the out-of-band power below specified limits. This leads to very inefficient amplification and expensive transmitters so that it is highly desirable to reduce the PAR [3]. Several schemes have been proposed to reduce the PAPR. These techniques can mainly be categorized into Signal scrambling techniques and Signal distortion techniques [1]. Signal scrambling techniques are all variations on how to scramble the codes to decrease the PAPR. Coding techniques can be used for signal scrambling. Golay complementary sequences, Shapiro-Rudin sequences, M-sequences, Barker codes can be used to efficiently reduce the PAPR. However, with the increase in the number of carriers, the overhead associated with exhaustive search of the best code would increase exponentially. More practical solutions of the signal scrambling techniques are block coding, selective mapping and partial transmit sequences.

The signal scrambling techniques can further be classified into:

- Schemes with explicit side information
  - Block codes: e.g. linear block code scheme, cyclic code scheme
  - Probabilistic schemes: e.g. SLM, PTS, Interleaving schemes
- Schemes without side information

Examples: Block coding scheme, Hadamard transform method, Dummy sequence insertion method etc.

Signal scrambling techniques, with side information reduces the effective throughput since they introduce the redundancy. The signal distortion techniques introduce both in-band and out-of-band interference and complexity to the system.

The signal distortion techniques reduce high peaks directly by distorting the signal prior to amplification. Clipping the OFDM signal before amplification is a simple method to limit PAPR. However, clipping ay cause large out-of-band (OOB) and in-band interference, which results in the system performance degradation. More practical solutions are peak windowing, peak cancellation, peak power suppression, weighted multi-carrier transmission, companding etc.

Basic requirement of practical PAPR reduction techniques include the compatibility with the family of existing modulation schemes, high spectral efficiency, and low complexity.

In this paper, a novel scrambling algorithm based on the use of Huffman coding, to reduce the PAPR significantly, is proposed. The paper is organized as follows: the PAR of OFDM signal is discussed in section 2, while Huffman Encoding is explained in section 3, simulation results are presented in section 4 and section 5 concludes the paper.

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II. PEAK TO AVERAGE RATIO

OFDM signals have a higher peak-to-average ratio (PAR) often called a peak-to-average power ratio (PAPR) than single-carrier signals do. The reason is that in the time domain, a multicarrier signal is the sum of many narrowband signals. At some time instances, this sum is large and at other times is small, which means that the peak value of the signal is substantially larger than the average value. This high PAR is one of the most important implementation challenges that face OFDM, because it reduces the efficiency and hence increases the cost of the RF power amplifier, which is one of the most expensive components in the radio. In this section, we quantify the PAR problem; explain its severity in OFDM systems.

A. The PAR Problem

When transmitted through a nonlinear device, such as a high-power amplifier (HPA) or a digital to analog converter (DAC) a high peak signal, generates out-of-band energy (spectral regrowth) and in-band distortion (constellation tilting and scattering). These degradations may affect the system performance severely. The nonlinear behavior of an HPA can be characterized by amplitude modulation/amplitude modulation (AM/AM) and amplitude modulation/phase modulation (AM/PM) responses. Figure (1) shows a typical AM/AM response for an HPA, with the associated input and output back-off regions (IBO and OBO, respectively).

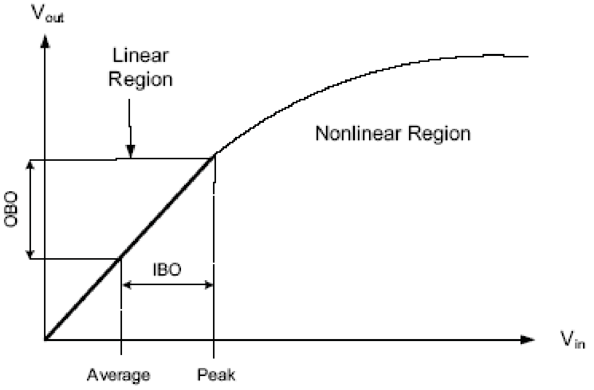

Figure (1) A typical power amplifier response.

To avoid such undesirable nonlinear effects, a waveform with high peak power must be transmitted in the linear region of the HPA by decreasing the average power of the input signal. This is called (input) *backoff* (**IBO**) and results in a proportional output backoff (**OBO**). High backoff reduces the power efficiency of the HPA and may limit the battery life for mobile applications. In addition to inefficiency in terms of power, the coverage range is reduced, and the cost of the HPA is higher than would be mandated by the average power requirements. The input backoff is defined as: [4]

$$IBO = 10 \log_{10} \frac{P_{in\,sat}}{\overline{P_{in}}}$$

Where P_{insat} is the saturation power, above which is the nonlinear region, and $\overline{P_{in}}$ is the average input power. The amount of backoff is usually greater than or equal to the PAR of the signal. The power efficiency of an HPA can be increased by reducing the PAR of the transmitted signal. Clearly, it would be desirable to have the average and peak values are as close together as possible in order to maximize the efficiency of the power amplifier. In addition to the large burden placed on the HPA, a high PAR requires high resolution for both the transmitter's DAC and the receiver's ADC, since the dynamic range of the signal is proportional to the PAR. High-resolution D/A and A/D conversion places an additional complexity, cost, and power burden on the system.

B. Quantifying the PAR

Since multicarrier systems transmit data over a number of parallel-frequency channels, the resulting waveform is the superposition of ***L*** narrowband signals. In particular, each of the ***L*** output samples from an ***L***-pt IFFT operation involves the sum of ***L*** complex numbers. Because of the Central Limit Theorem, the resulting output values $\{x_1, x_2, \ldots, x_L\}$ can be accurately modeled, particularly for large ***L***, as complex Gaussian random variables with zero mean and variance $\sigma^2 = \varepsilon_x / 2$; that is the real and imaginary parts both have zero mean and variance $\sigma^2 = \varepsilon_x / 2$. The amplitude of the output signal is:

$$|x[n]| = \sqrt{(\mathrm{Re}\{x[n]\})^2 + (\mathrm{Im}\{x[n]\})^2}$$

Which is Rayleigh distributed with parameter σ^2. The output power is therefore

$$|x[n]|^2 = (\mathrm{Re}\{x[n]\})^2 + (\mathrm{Im}\{x[n]\})^2$$

And thus the output power is exponentially distributed with mean $2\sigma^2$. The important thing is that the output amplitude and hence power are ***random***, so the PAR is not a deterministic quantity. The PAR is considered for a single OFDM symbol, which consists of $L+N_g$ samples, the discrete-time PAR can be defined for the IFFT output as: [1-4]

$$PAR \hat{=} \frac{\max |x_l|^2}{E[|x_l|^2]} \quad , l \in (0, L + N_g)$$

It is important to recognize, however, that although the average energy of IFFT outputs is the same as the average energy of the inputs and equal to ε_x, the analog PAR is ***not*** generally the same as the PAR of the IFFT samples, owing to the interpolation performed by the D/A converter. Usually, the analog PAR is higher than the digital (*Nyquist sampled*) PAR. Since the PA is by definition analog, the analog PAR is what determines the PA performance. In order to bring the analog PAR and the digital PAR closer together, over sampling can be considered for the digital signal. That is, factor ***M*** additional samples can be used to interpolate the digital signal in order to better approximate its analog PAR.

Since the theoretical maximum (or similar) PAR value seldom occurs, a statistical description of the PAR is commonly used. The Complementary Cumulative Distribution Function (CCDF = 1 – CDF) of the PAR is the most commonly used measure. The distribution of the OFDM PAR has been studied by many researchers. Among them, Van Nee and de Wild in [5] introduced a simple and accurate approximation of the CCDF for large ***L*** (≥64):

$$CCDF(L,\varepsilon_{\max}) = 1 - F(L,\varepsilon_{\max})^{\beta L}$$

$$= 1-(1-\exp(\frac{\varepsilon_{\max}}{2\sigma^2}))^{\beta L}$$

Where $\varepsilon_{\max}$ is the peak power level and β is a pseudo approximation of the over sampling factor, which is given empirically as $\beta = 2.8$. Note that the PAR is $\varepsilon_{\max}/2\sigma^2$ and $F(L,\varepsilon_{\max})$ is the Cumulative Distribution Function (CDF) of a single Rayleigh-distributed subcarrier with parameter σ^2. Figure (2) shows a plot of the CCDF of the PAR for OFDM systems with different Number of subcarriers (L=64, 128, 256, 512), it is clear that the PAR increases as L increase but not nearly linearly.

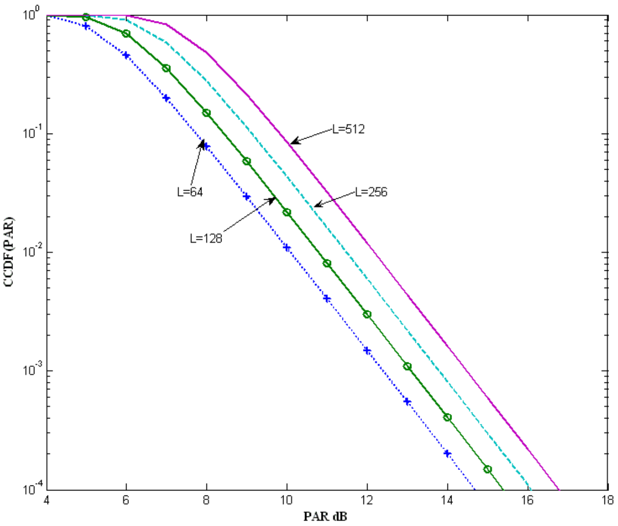

Figure (2) CCDF (PAR) for different L values

III. HUFFMAN ENCODING

This method is named after D.A. Huffman, who developed the procedure in the 1950s. The idea is to assign frequently used signal sample values fewer bits, and seldom used sample values more bits to make an appropriate compression for the signal to be transmitted. In mathematical terms, the optimal situation is reached when the number of bits used for each sample is proportional to the logarithm of the sample's probability of occurrence [6]. A clever feature of Huffman encoding is how the variable length codes can be packed together. Imagine receiving a serial data stream of ones and zeros. If each character is represented by eight bits, you can directly separate one character from the next by breaking off 8 bit chunks. Now consider a Huffman encoded data stream, where each character can have a variable number of bits. How do you separate one character from the next? The answer lies in the proper selection of the Huffman codes that enable the correct separation. An example will illustrate how this works: The characters *A* through *G* occur in the original data stream with the probabilities shown in table (1). Since the character *A* is the most common, we will represent it with a single bit, the code: 1. the next most common character, *B*, receives two bits, the code: 01. This continues to the least frequent character, *G*, being assigned six bits, 000011. As shown in this illustration, the variable length codes are resorted into eight bit groups.

Table (1) Example Encoding Table

Letter	Probability	Huffman Code
A	0.154	1
B	0.110	01
C	0.072	0010
D	0.063	0011
E	0.059	0001
F	0.015	000010
G	0.011	000011

When uncompressing occurs, all the eight bit groups are placed end-to-end to form a long serial string of ones and zeros. Look closely at the encoding table (1), and notice how each code consists of two parts: a number of zeros before a *one*, and an optional binary code after the *one*. This allows the binary data stream to be separated into codes without the need for delimiters or other marker between the codes.

To implement Huffman, the compression and uncompression algorithms must agree on the binary codes used to represent each character (or groups of characters). This can be handled in one of two ways. The simplest is to use a predefined encoding table that is always the same, regardless of the information being compressed. More complex schemes use encoding optimized for the particular data being used. This requires that the encoding table be included in the compressed file for use by the uncompression program. Both methods are common. In this paper we have chosen the second method, this implies that we will send the encoding table as side information but this will not affect the bandwidth efficiency because the compression gained by the Huffman coding will counteract the increase in data rate due to the side information transmitted.

Huffman coding can be used in the OFDM system as shown in figure (3):

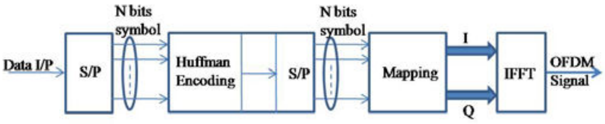

Figure (3) OFDM transmitter with Huffman Coding

The input data stream is converted from serial to parallel among M symbols (each of n bits), while M=2^n, which is Huffman encoded yielding a serial bit stream, that is again converted to parallel among M symbols (each of n bits also), not to mention that these M symbols have different probability distribution, then mapped to IQ form according to the modulation format used and finally the OFDM composite time signal is produced through the IFFT stage.

When applied to the OFDM signal, the Huffman encoding will cause the PAR to be reduced, that is because of its nature as mentioned above, that the encoding causes the frequently occurring symbols is assigned a lower number of bits rather than the less probable to occur symbols, and thus when rearranging the stream of bits among symbols with fixed number of bits , the probability of repeating the same symbol will be eliminated, preventing the coherent addition of the multicarrier signals that cause the undesired very high peak.

IV. SIMULATION RESULTS

A. System model

In this paper, the simulated system employs an OFDM signal with 16-QAM modulated, N=256 subcarriers and oversampling factor of 2, i.e. IFFT length= 512. The HPA is Rapp's solid state power amplifier with the characteristic: [7]

$$v_{out} = \frac{v_{in}}{(1+(|v_{in}|/v_{sat})^{2p})^{\frac{1}{2p}}}$$

Where v_{out} and v_{in} are complex i/p & o/p, v_{sat} is the output at the saturation point and p is the smoothness factor during its transition from the linear region to the saturation region, figure(4) illustrates the AM/AM curves for a few different smoothness factors, p. A typical value of $p=3.58$ is given in [7].

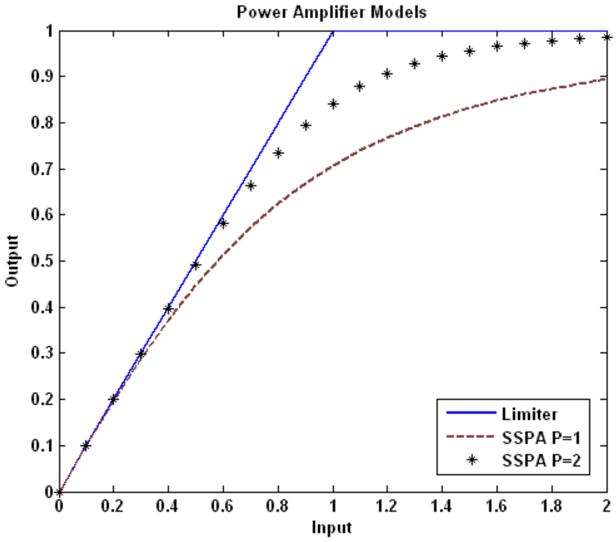

Figure (4) AM/AM curves for HPA (p=1, 2)

We have chosen two distinct signals, firstly; we have chosen a 128*128 Gray image and transformed it to an 8-bit levels. Thus the number of bits of the Uncoded signal equals 8*128*128=131072 bits, when designing a Huffman code for this image: we've got a coded signal of 128828 bits, the reduction in bit rate for this image is 2244 can be used to send the encoding table as side information. The second signal is a 2 sec speech signal (64 kbps).

When applying the two signals to the proposed system model we've found that the use of Huffman coding reduces the PAR significantly with about 6 dB for CCDF(PAR) $\leq 10^{-2}$ and about 4 dB for $10^{-2} \leq$ CCDF(PAR) $\leq 10^{-1}$ as depicted in figure(5) where the CCDF of PAR for the OFDM system with and without Huffman coding is plotted.

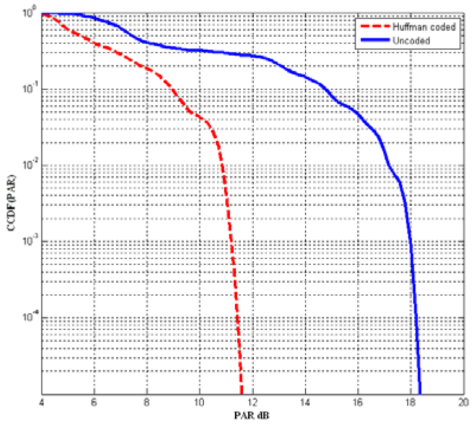

Figure (5) CCDF(PAR) with /without Huffman coding

The two primary drawbacks from nonlinear amplification are (1) spectral regrowth, frequency domain leakage, which causes unacceptable interference to users in neighboring RF channels which is referred to as Adjacent Channel Power Ratio (ACPR), and (2) distortion of the desired signal or what is conventionally called In-Band distortion which is measured by both the Error vector magnitude EVM and the BER. We now consider these two effects separately.

First, we will check on the effect of our proposed solution for reducing the spectral regrowth due to the HPA nonlinearities through plotting the OFDM signal spectrum for different power amplifier IBO values we got that shown in figure (6)

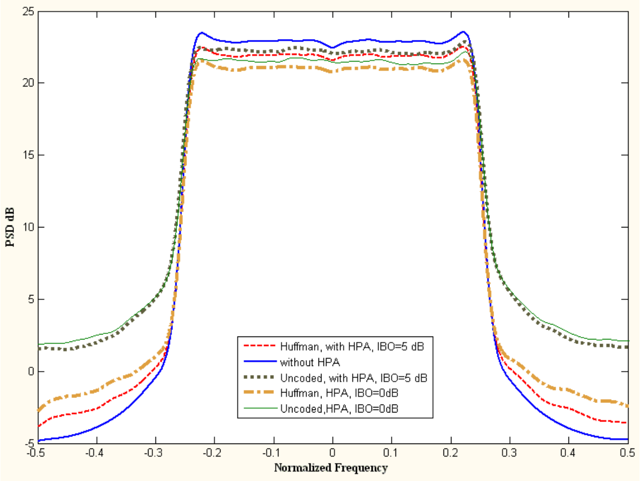

Figure (6) OFDM spectrum with HPA

It is clear to note from figure (6) that the use of Huffman coding reduces greatly the spectral regrowth due to the HPA with IBO= 5 dB as compared to the case of uncoded signal, it is also noted that when applying HPA with lower IBO (=0dB), the spectral regrowth in the case of Huffman coded signal is larger than that for IBO=5 dB where the spectral regrowth in the case of uncoded signal will be nearly the same for both IBO=0dB and 5dB. This is logically agree with the CCDF plot in figure (5) where in the case of uncoded signal, the CCDF(PAR) $\approx$ 1 for PAR $\leq$ 5dB, this implies that for all values of IBO less than 5 dB, the effect of HPA on the OFDM signal will be the same. This is not the case in the Huffman coded signal.

As we mentioned before, the spectral regrowth can be quantified by the ACPR, table (2) shows the effect of HPA with different IBO values on the ACPR for both uncoded and Huffman coded signals, it can be noted that as the IBO value decreases the ACPR increase, also the effect of HPA on the Huffman coded signal is less than that on the uncoded signal for the same IBO value, in a complete agreement to that concluded from figure (6).

Table (2) shows also the effect of HPA on the EVM for both Huffman coded and uncoded signals for different IBO values, as a measure of the inband distortion as we mentioned before. It is also confirming our conclusion that the effect of HPA on the Huffman coded signal is less than that on the uncoded signal for the same IBO value.

Table (2) Effect of HPA on OFDM with different IBO value

IBO	Uncoded signal		Huffman Coded	
	ACPR	EVM	ACPR	EVM
No HPA	- 25 dB		-25 dB	
6 dB	- 20 dB	0.4365	-25 dB	0.245
5 dB	- 18 dB	0.4589	-23 dB	0.2734
0 dB	- 17.5dB	0.58	-21 dB	0.4390

Also when monitoring the BER for different IBO's of the HPA in the two mentioned cases, we note as in figure (7)

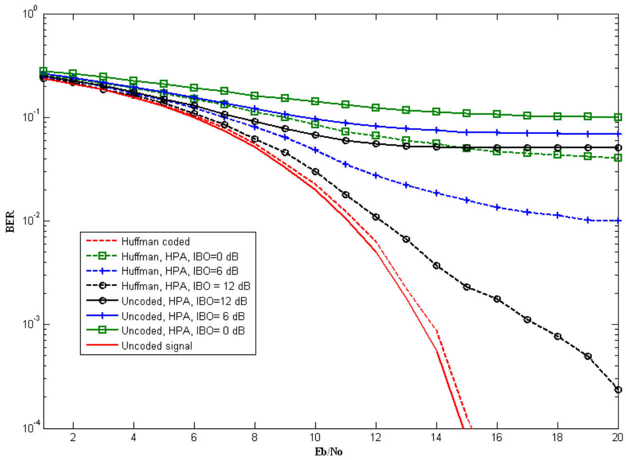

Figure (7) BER for different IBO's of the HPA (0, 6, 12 dB)

It is obvious to note that there is a noticeable immunity to the HPA when using Huffman coding with the OFDM system as compared to the case of uncoded signal. Also it is clear that the effect of HPA is nearly the same when applied to the OFDM system without Huffman coding with different IBO values, that the BER curve of IBO = 0 dB is very close to that of IBO = 12 dB, this remark is logically expected from figure (4) that the CCDF > 10^{-1} for the Uncoded signal for PAR<15, while this situation is different from that of the Huffman coded signal, that the HPA with IBO = 12 dB has slight effect on the BER curve, but the HPA with IBO = 0 dB has nearly the same effect as on the Uncoded signal.

V. CONCLUSION

We have concluded that the use of Huffman coding greatly reduce the PAR of the OFDM system (by about 6 dB) and consequently reduces the effect of nonlinearities due to HPA, particularly, the spectral spreading and inband distortion.

Also, as the IBO of the HPA decrease, the spectral spreading and inband distortion increase.

For the uncoded signal, the distortion due to HPA with IBO= 0 dB is very close to that of HPA with IBO = 12 dB.

For both the Uncoded and Huffman coded signal the HPA with IBO = 0 dB is nearly the same.

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