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Perceptual Framework for a Modern Left-Turn Collision Warning System

E. Dabbour and S. M. Easa

***Abstract*—**Most of the collision warning systems currently available in the automotive market are mainly designed to warn against imminent rear-end and lane-changing collisions. No collision warning system is commercially available to warn against imminent turning collisions at intersections, especially for left-turn collisions when a driver attempts to make a left-turn at either a signalized or non-signalized intersection, conflicting with the path of other approaching vehicles traveling on the opposite-direction traffic stream. One of the major factors that lead to left-turn collisions is the human error and misjudgment of the driver of the turning vehicle when perceiving the speed and acceleration of other vehicles traveling on the opposite-direction traffic stream; therefore, using a properly-designed collision warning system will likely reduce, or even eliminate, this type of collisions by reducing human error. This paper introduces perceptual framework for a proposed collision warning system that can detect imminent left-turn collisions at intersections. The system utilizes a commercially-available detection sensor (either a radar sensor or a laser detector) to detect approaching vehicles traveling on the opposite-direction traffic stream and calculate their speeds and acceleration rates to estimate the time-to-collision and compare that time to the time required for the turning vehicle to clear the intersection. When calculating the time required for the turning vehicle to clear the intersection, consideration is given to the perception-reaction time of the driver of the turning vehicle, which is the time required by the driver to perceive the message given by the warning system and react to it by engaging the throttle. A regression model was developed to estimate perception-reaction time based on age and gender of the driver of the host vehicle. Desired acceleration rate selected by the driver of the turning vehicle, when making the left-turn movement, is another human factor that is considered by the system. Another regression model was developed to estimate the acceleration rate selected by the driver of the turning vehicle based on driver's age and gender as well as on the location and speed of the nearest approaching vehicle along with the maximum acceleration rate provided by the mechanical characteristics of the turning vehicle. By comparing time-to-collision with the time required for the turning vehicle to clear the intersection, the system displays a message to the driver of the turning vehicle when departure is safe. An application example is provided to illustrate the logic algorithm of the proposed system.

***Keywords*—**Collision warning systems, intelligent transportation systems, vehicle safety.

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I. Introduction

LEFT-TURN collisions are among the most severe types of collisions that result in high rates of fatalities and injuries [1]. They typically occur when a driver attempts to make a left-turn from either an uncontrolled road or a signalized road during the permitted green phase while other vehicles are approaching the intersection from the opposite direction. A major factor that leads to left-turn collisions at intersections is driver's misjudgment of the speed and acceleration of the vehicles traveling on the opposite direction [2]. This human inadequacy is profound in all drivers with some variations among drivers due to age, health conditions and other factors. Although the human visual system is extremely sophisticated, it has no direct perceptual mechanisms to support the perception of acceleration [3]. A properly-designed collision warning system might mitigate that problem by detecting all approaching vehicles, measuring their speeds and acceleration rates, and giving a visual, auditory, or haptic signal to the driver of the host vehicle to start left-turn movement when a safe departure is warranted.

Many research projects were aimed at designing vehicle-mounted intersection collision warning systems, including the Intersection Collision Avoidance (ICA) system developed by Calspan SRL Corporation [2], the Intersection Crash Avoidance, Violation warning (ICAV) system proposed by Virginia Tech Transportation Institute [4], and the INTERSAFE system developed by the European Commission [5]. Each of those systems utilized a pair of detectors (either radar sensors for ICA and ICAV systems or laser scanners for INTERSAFE system) installed at the left and right front corners of the host vehicle to detect cross-traffic vehicles at intersections and determine their speeds and time-to-collision and trigger a warning if they found to be conflicting with the path of the host vehicle. The left-side detector was also utilized in detecting vehicles in the opposite-direction traffic stream, in situations when the host vehicle makes a left turn, so that their speeds and time-to-collision can be determined to trigger an appropriate warning if the left-turning movement is not safe. However, all of those research projects have some limitations in terms of the lack of consideration given to measuring the acceleration of the detected vehicles as well as the lack of consideration given to the time required for the driver of the host vehicle to perceive the messages given by the warning system and react to it. The desired acceleration rate selected by the driver of the host vehicle, when departing

the intersection, is another human factor that was not considered by those previous research projects.

Another infrastructure-based collision-warning system was proposed for left-turning vehicles [6]. The system was aimed to providing guidance to left-turning motorists in the form of a dynamic sign placed at the same line of sight of the opposite-direction traffic, so that the driver of the turning vehicle would have a "second opinion" to aid in his/her left-turn decision. That infrastructure-based system has some limitations. The system assumes average values for the mechanical characteristics of the turning vehicle in terms of size and acceleration. The system also did not give consideration for the perception-reaction time of the driver who makes the left turn. The time-to-collision for the approaching vehicle was calculated by assuming that the oncoming vehicles have a constant speed with no acceleration. In addition, there are legal liability issues that might arise as a result of using an infrastructure-based system.

This paper gives perceptual framework for a proposed technology-independent collision warning system for left-turning vehicles at intersections by utilizing a detection sensor (either a radar sensor or a laser scanner) installed at the left-side front corner of the host vehicle. Unlike other previously proposed systems, this system's algorithm gives consideration for the time required for the driver of the host vehicle to perceive the message given by the system and react to it. The algorithm also gives consideration for the variations among drivers in selecting their desired acceleration rates when making a left-turn movement. An application example is also presented to illustrate the logic algorithm of the proposed system.

II. PROPOSED WARNING SYSTEM

Similar to previously proposed intersection collision warning systems, the proposed system utilizes a detection sensor (either a radar sensor similar to the ICA and ICAV systems, or a laser scanner similar to the INTERSAFE system) that is installed at the left-side front corner of the host (*target*) vehicle to detect approaching (*bullet*) vehicles in the opposite-direction traffic stream as shown in Fig. 1. Using the detection sensor, along with a processing unit and driver-vehicle interface, the system determines whether a '*Safe*' or '*Not Safe*' message should be displayed to the driver of the host (*target*) vehicle

A. Detection Sensor's Minimum Specifications

Detection sensor sends a beam every time interval, t, to scan the traffic lanes in the opposite direction of the road. The time interval ranges from 0.04 sec to 1.5 sec, depending on the type of the detector used. From previous research [4] an update rate of 10 Hz, with 0.1 second time interval is recommended to provide acceptable range and range-rate resolution. The width of the beam is designed so that the angle between the outer left edge of the beam and the face plane of the vehicle, Φ_1, is within the range (10°-22°), as shown in Fig. 1. This is to ensure no opposite vehicle, which may potentially collide with the turning vehicle, is outside the coverage area of the radar beam. The maximum azimuth angle, Φ_2, is within the range (55°-70°) to cover three lanes in the opposite direction in addition to a median.

Based on previous research [4], [7] and [8] the other required specifications for the detection sensor used are as follows:

1. Maximum range no less than 150 m (to detect all vehicles traveling in the opposite direction that may collide with the host vehicle).
2. Opening (azimuth) angle (horizontal) 55° - 70° (as explained above).
3. Opening (azimuth) angle (vertical) 4° - 8°
4. Minimum resolution of azimuth angle 0.1°
5. Maximum data latency 0.05 sec
6. Maximum range-rate no less than 30 m/sec (assuming maximum speed on the opposite direction of the major road of 108 km/h).
7. Range resolution no less than 0.05 m (given the small cycle time).
8. Update rate is 10 Hz, with 0.1-sec time interval.

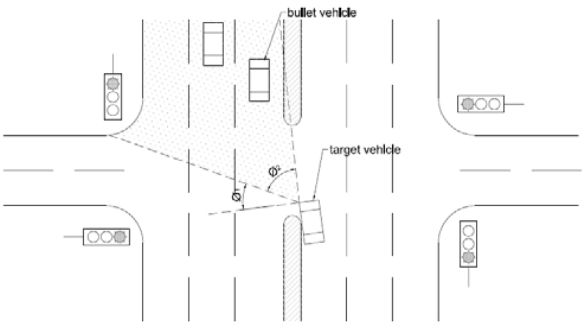

Fig. 1 System configuration

The above are technology-independent minimum specifications and further research will be required to select the product that meets them. Possible candidates may include the following detection sensors:

1. The EVT-200 radar sensor produced by Eaton VORAD Technologies [9]. That sensor was utilized by the ICA system [2]; however, the maximum opening angle provided by that sensor is smaller than required and technical modifications may be required to increase the opening angle.
2. The UMRR-007xx radar sensor produced by Smart Microwave Sensors [10]. That sensor may meet the minimum specifications required; however, given that it is fully customized, its price may be too high to be implemented for commercial use.
3. The LUX laser scanner produced by ibeo Automobile Sensor [11]. That sensor was emerged from the original ALASCA XT scanner that was utilized by the INTERSAFE system (Fuerstenberg and Chen 2007); however, further research may be required to

ensure its functionality under different weather conditions.

B. System Algorithm

The algorithm procedures are as follows (Fig. 2):

1. Once the brakes are activated and the detection sensor detects moving objects within its coverage area (the opposite traffic lanes) the system displays a warning message to the driver by default. That message could be visual, auditory, haptic, or a combination of two methods. The message is not deactivated until the algorithm confirms that a safe left turn is available as per the subsequent steps;
2. The system estimates the time required for each detected vehicle, t_{bullet}, to reach the intersection;
3. The system also estimates the time required for the turning vehicle, t_{target}, to complete the left-turn movement;
4. The system compares t_{bullet} and $t_{target,}$ and makes a decision as per the following criterion:
    a. If t_{target} is found to be less than t_{bullet}, the warning message is deactivated and a '*Safe*' message is displayed to the driver of the turning vehicle to allow him/her to start the left-turn movement; or
    b. If t_{target} is found to be greater than or equal to t_{bullet}, the warning message continues to be active and the system repeats the algorithm until the criterion (a) above is met.

The proposed algorithm tracks different approaching vehicles on different lanes and the above procedures are followed for each vehicle with no '*safe*' message displayed until all lanes are clear of approaching vehicles that may collide with the turning vehicle. Similar to preceding ICA, ICAV and INTERSAFE systems, vehicle tracking is performed by using a Kalman filter [12]. The Kalman filter performs tracking by estimating the state of dynamic objects (i.e., the approaching vehicles) at different time intervals from a series of incomplete and noisy measurements (taken by detection sensor) and provides accurate continuously-updated information about the position and speed of those approaching vehicles. A bounding box is placed around the predicted positions of every approaching vehicle and logic is used to determine if the detection is within that bounding box and hence is associated with a specific track. More information about vehicle tracking using Kalman filter can be found in literature such as [2], [13] and [14]. It should be noted that generating a '*safe*' message is based on the assumption that the intersection is adequately designed for intersection sight distance as given by design guides and previous research [15] and [16].

C. Bullet Vehicle Time

To calculate the time required for a detected *bullet* vehicle to reach the intersection, t_{bullet}, a detection beam is generated at time T to scan traffic lanes at opposite direction. If no object is detected, a '*safe*' signal is displayed to the driver. Otherwise, the nearest vehicle detected, vehicle '*A*', is registered at range d_1 and azimuth angle θ_1 where polar coordinates are used with the origin point coincides with the location of the detection sensor as shown in Fig. 3. Another detection beam is generated at time $T+t$, where t is the time interval for the detector, and the new location of vehicle '*A*' is registered at range d_2 and azimuth angle θ_2 where the same above polar coordinates are used, as shown in Fig. 4. If d_1 was found to be equal to d_2, the algorithm concludes that the object is not moving (e.g., a tree or a building), and a '*safe*' message is displayed to the driver (unless another object is detected on another lane). If d_2 was found to be greater than d_1, the algorithm concludes that the object is moving away from the host (*target*) vehicle. A '*safe*' message is also displayed to the driver (unless another object is detected). Finally, if d_2 was found to be less than d_1, the speed of the approaching vehicle during the first time interval, v_1, is calculated by the following formula (as illustrated in Fig. 5):

$$v_1 = \frac{\sqrt{d_1^2 + d_2^2 - 2d_1 d_2 \cos(\theta_1 - \theta_2)}}{t} \quad (1)$$

Where the term under the square root is the distance traversed by the *bullet* vehicle during the first time interval (denoted as d_{v1}). Similarly, a third radar beam is generated at time $T+2t$ and the range, d_3, and azimuth angle, θ_3, are registered. The vehicle speed during the second time interval is given by:

$$v_2 = \frac{\sqrt{d_2^2 + d_3^2 - 2d_2 d_3 \cos(\theta_2 - \theta_3)}}{t} \quad (2)$$

The vehicle acceleration, a, is calculated using the formula:

$$a = \frac{v_2 - v_1}{t} \quad (3)$$

From Fig. 6, the side offset between the *bullet* vehicle and the *target* vehicle, w_f, is calculated as:

$$w_f = d_3 \sin\beta = d_3 \frac{d_2}{d_{v2}} \sin(\theta_2 - \theta_3) \quad (4)$$

The distance from the *bullet* vehicle '*A*' (at time $T+2t$) to the intersection, d_f, is calculated as:

$$d_f = \sqrt{d_3^2 - w_f^2} \quad (5)$$

The anticipated speed where the bullet vehicle is anticipated to reach the intersection, v_f, is calculated using the formula:

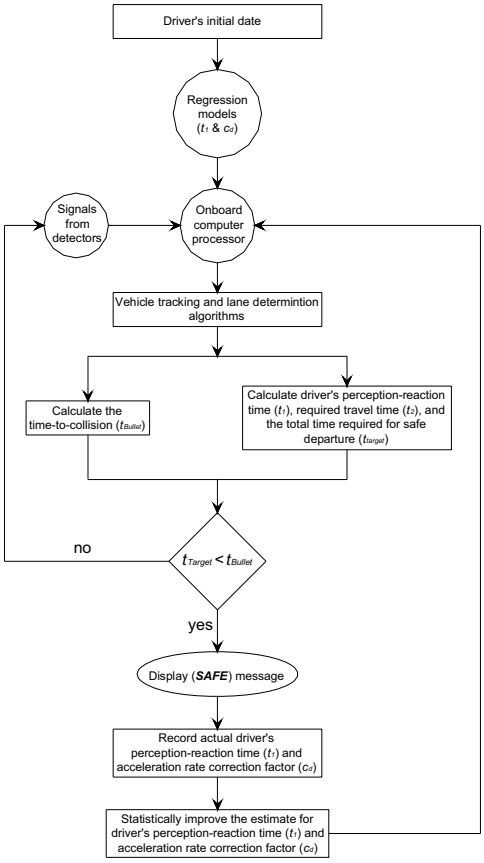

Fig. 2 Flow chart for the proposed algorithm

$$v_f = \sqrt{v_2^2 + 2ad_f} \tag{6}$$

The time required for the bullet vehicle to reach the intersection, t_{bullet}, is then given by

$$t_{bullet} = \frac{v_f - v_2}{a} \tag{7}$$

Where v_2 is calculated from Equation 2, a is calculated from Equation 3, and v_f is calculated from Equation 6.

D. Target Vehicle Time

The total time required for the driver of the *target* vehicle to clear the intersection, t_{target}, is given by

$$t_{target} = t_1 + t_2 \tag{8}$$

Where t_1 is the perception-reaction time for the driver of the host vehicle and t_2 is the crossing time. The following sections discuss the procedures followed to calculate both t_1 and t_2.

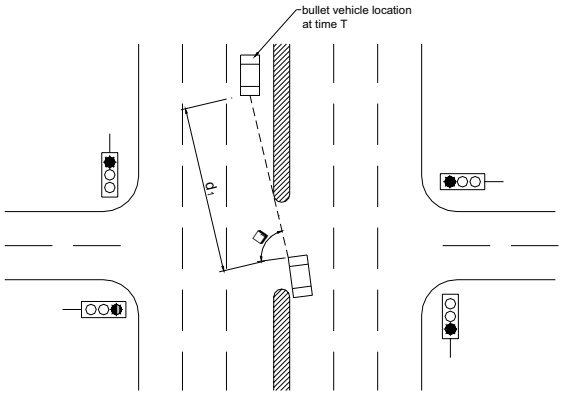

Fig. 3 Distance and azimuth angle at times T

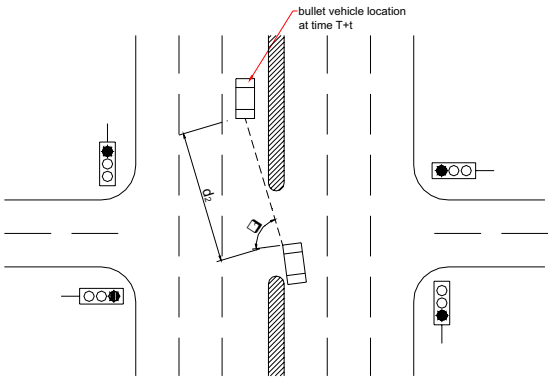

Fig. 4 Distance and azimuth angle at times $T + t$

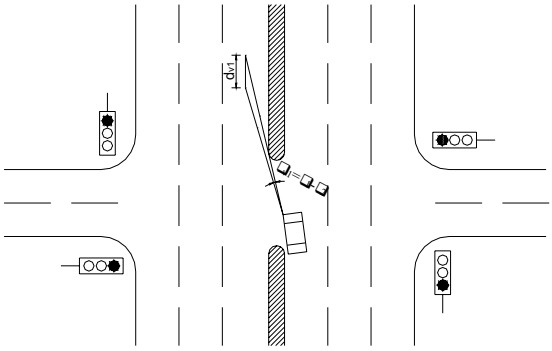

Fig. 5 Distance traversed during the first time interval

E. Perception-reaction Time

The driver's perception-reaction time, t_1, is different for the cases when a collision warning system is and is not installed. If a collision warning system is installed, the perception-reaction time for the driver is the time required to perceive the message given by the collision warning system and to react

accordingly by activating throttle. On the other hand, if a collision warning system is not installed, the driver has to perceive the whole situation, analyze it, make a decision and take appropriate action by activating the throttle. Theoretically, the perception-reaction time if a collision warning system is installed is expected to be less than that if no collision warning system is installed due to the fewer mental tasks performed by the driver (no analysis or decision-making required). However, this expectation is influenced by many factors, including the reliability of the warning system and driver's familiarity with it; therefore, a regression model was developed to estimate driver's perception-reaction time (t_1) as a dependent variable based on driver's gender and age as independent variables. The model was developed using data collected from 60 drivers representing both genders and different age groups as shown in Table I. The sample was selected to proportionally represent the population of licensed drivers in Canada in 2003 [17].

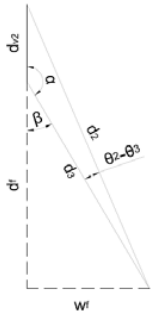

Fig. 6 Distance d_f and offset w_f

Every driver was asked to drive three driving simulation scenarios on the STISIM driving simulator [18] located at Ryerson University (Toronto, Canada). The scenarios were designed to simulate a series of intersections where drivers were asked to perform a left-turn movement at each of them. The algorithm was encoded into the scenarios so that at each intersection the driver would hear a buzz signal indicating that it was not safe to depart the intersection. The driver was instructed to depart the intersection as soon as the buzz signal stops for that intersection. The data collected included the perception-reaction time for the driver from the time the buzz signal stops to the time when he/she starts to engage the throttle to complete the required left-turn departure movement. A total of 3600 observations were used to calibrate the following regression model:

$$t_1 = 0.2466 + 0.0241(AGE) + 0.1353(GENDER) \quad (9)$$

Where t_1 is the perception-reaction time for the driver, *AGE* is the age of the driver (in years), and *GENDER* is a dummy variable that represents driver's gender (0 for male and 1 for female). The coefficients of all independent variables were found to be significantly different from zero at the 95% confidence level. The coefficient of regression, R^2, was found to be 0.87, and the standard deviation was found to be 0.54 seconds. As a conservative approach, the perception-reaction time (estimated by Equation 9) may be corrected by adding the standard deviation.

TABLE I
SAMPLE SELECTION AND LICENSED DRIVERS IN CANADA

Age Group	Sample Size			Licensed Drivers In Canada	
	Male	Female	Percentage	Number of Licensed Drivers	Percentage
15-19	2	1	5.00%	1,087,986	5.08%
20-24	3	2	8.33%	1,754,394	8.19%
25-34	6	5	18.33%	3,833,556	17.89%
35-44	7	7	23.33%	4,760,515	22.22%
45-54	6	5	18.33%	4,358,434	20.34%
55-64	4	4	13.33%	2,924,581	13.65%
65+	4	4	13.33%	2,707,821	12.64%
Total	32	28	100.00%	21,427,287	100.00%

F. Driver's Acceleration Rate

The acceleration rate selected by the driver is the product of the maximum acceleration rate provided by the mechanical characteristics of the vehicle (as given by the vehicle's performance data) multiplied by a correction factor, c_d, which depends on driver's characteristics as well as the distance and speed of the nearest approaching vehicle. A regression model was calibrated from observations taken from the same driver sample used for calibrating the perception-reaction time model. The model is given by:

$$c_d = 0.95164 - 0.00228(AGE) - 0.01976(GENDER) - 0.00517(d_f) + 0.02325(v_2) \quad (10)$$

Where c_d is the driver's correction factor for the acceleration rate, d_f is the distance (in meters) to the nearest detected approaching vehicle (as was calculated from Equation 5), v_2 is the detected speed (in meter per second) of the nearest approaching vehicle during the last time interval used by the algorithm (as was calculated from Equation 2), and the variables *GENDER* and *AGE* are the same as in the model for perception-reaction time. The coefficients of all independent variables were significantly different from zero at the 95% confidence level. The signs of all the variables are logical. The driver's acceleration rate, a_d, is calculated from the following simple formula:

$$a_d = a_v x c_d \quad (11)$$

Where c_d is calculated from Equation 10 and a_v is the maximum acceleration rate provided by the mechanical characteristics of the vehicle as given by the vehicle's performance data that are pre-loaded to the algorithm.

G. Vehicle Travel Time

Vehicle travel time, t_2, is the time required for the vehicle to accelerate and to clear the path of the approaching vehicles on the opposite traffic lanes. The total distance to be crossed by the *target* vehicle in this case is given by:

$$S = w_f + L \tag{12}$$

Where S is the total distance to be crossed, w_f is the side offset between the target vehicle and the bullet vehicle (as calculated from Equation 4), and L is the length of the turning vehicle. The time required for crossing, t_2, is then given by the following equation (assuming that the initial speed is zero):

$$t_2 = \frac{v}{a_d} = \frac{\sqrt{2a_d S}}{a_d} = \sqrt{\frac{2S}{a_d}} \tag{13}$$

Where S is calculated from Equation 12 and a_d is calculated from Equation 11.

H. Decision Making

The final step of the algorithm is to compare the time required for the *bullet* vehicle to reach the intersection, t_{bullet}, with the time required for the *target* vehicle to depart the intersection, t_{target}. A factor of safety may be added to represent a minimum gap between the *target* and *bullet* vehicles. From previous research [19] it was found that the majority of left-turning drivers feel more comfortable when the gap between their turning vehicle and other vehicles approaching on the opposite traffic lanes is 2 seconds; and therefore, to keep the proposed system consistent with driver's normal behavior (so that the system does not cause nuisance to the driver), the following criterion is used:

$$t_{target} - t_{bullet} > 2.0 \text{ sec.} \tag{14}$$

If the above criterion is met for all detected vehicles, a '*safe*' message is displayed to the driver; otherwise, the '*not safe*' message continues to be displayed.

III. APPLICATION EXAMPLE

The following application example is provided to illustrate the algorithm procedures. Assume a host *target* vehicle with a length of 4.2 m and a maximum rate of acceleration of 5.25 m/sec^2. The driver of the vehicle is 32 years old male. Using a detection sensor with a cycle time of 0.5 sec (update rate 2 Hz), three consecutive readings for the range of an approaching vehicle, on the opposite traffic lanes, were found to be 140.45 m, 132.50 m, and 124.45 m, respectively. The corresponding azimuth angle readings were 85.1°, 84.8°, and 84.5°, respectively.

From Equations 1 and 2, the speed of the approaching vehicle during the first and second time intervals is found to be 15.96 m/sec. and 16.16 m/sec., respectively. The acceleration rate of the approaching vehicle is 0.4 m/sec^2, and the offset between the approaching and the turning vehicles is 10.69 m. The distance from the approaching vehicle to the intersection (at the end of the third cycle) is 123.99 m. The final speed where the approaching vehicle is estimated to reach the intersection is 18.98 m/sec, and the time required for the approaching vehicle to reach the intersection, t_{bullet}, is 7.0 sec.

The perception-reaction time and driver's correction factor for the acceleration rate are both calculated from the regression models and found to be 1.0 sec and 0.6134, respectively. The total distance to be crossed by the turning vehicle is 14.89 m, and the required vehicle travel time, t_2, is therefore 3.0 sec. The total time required to depart the intersection, t_{target}, is 4.0 sec.; and therefore, since the difference between t_{target} and t_{bullet}, is more than 2.0 sec, the system should display a '*safe*' signal to the driver. It should be noted that if the driver of the turning vehicle is older, his perception-reaction time will be longer and therefore a '*Safe*' message might not be warranted under the same circumstances. In this example, a '*Safe*' message will not be warranted if the driver is 62 years or older. It should also be noted that the maximum acceleration rate of the host *target* vehicle is another important factor that affects its departure time. In this example, a '*safe*' message will not be warranted if the maximum acceleration rate for the host vehicle is 3.0 m/sec^2 or less.

IV. CONCLUSIONS

This paper proposed the perceptual framework for a collision warning system to detect left-turn collisions. The proposed warning system might mitigate collisions by precisely calculating the speeds and acceleration rates of all approaching vehicles and estimating the time-to-collision. By comparing that time with the time required for the turning vehicle to complete the left-turn movement, the warning system can advise the driver when departure is safe. Based on this research the following concluding remarks are offered:

1. The proposed warning system utilizes a commercially-available detection sensor (either a radar sensor or a laser detector) that is capable of recognizing different approaching vehicles on different lanes and no '*Safe*' signal is given to the driver of the turning vehicle until all lanes are clear of the approaching vehicles that could collide with the turning vehicle.
2. To determine the time required for the left-turn movement, a regression model was developed to estimate the perception-reaction time for the driver of the turning vehicle based on his/her age and gender. Another regression model was also developed to estimate the acceleration rate selected by the driver of the turning vehicle, when making the left-turn departure movement, based on different factors including age, gender, the speed and location of the nearest approaching vehicle as well as the maximum acceleration rate provided by the mechanical characteristics of the turning vehicle. The reliability of the warning system can be dramatically improved by recording the actual response

times and acceleration rates for the specific driver on a specific vehicle and creating a database to be used to update the regression models using appropriate mathematical techniques, such as artificial neural network analysis.

3. The described warning system assumes a constant rate of acceleration for the approaching vehicle. This assumption is justified by the fact that the maximum range for the radar sensor used is 150 m, which yields a maximum detected t_{bullet} of approximately 8 sec (for typical speeds of 60-70 km/h at urban intersections). Given the mechanical characteristics of most existing vehicles, it is likely that the rate of change in acceleration will be small and the resulting error in calculating t_{bullet} will be negligible. However, to further improve accuracy, a fourth radar reading may be required to estimate the rate of change in acceleration of the approaching vehicle.
4. Successful implementation of the proposed system would require the intersections to have adequate sight distances in order for the system to detect approaching vehicles; and therefore, the proposed system should not be considered as an aid to the driver in situations where intersection sight distance is not adequately provided. Note also that the scope of this paper is limited to the algorithm design for the proposed system and further research is required to address other aspects such as circuit design of the system and building a prototype to be tested with actual vehicles prior to introducing the system as a commercial product.
5. The developed left-turn warning system can be integrated with existing collision warning systems, which are mainly designed to detect rear-end and lane-changing collisions. It is hoped that the proposed warning system will reduce left-turn collisions and help improve safety at intersections.

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