Adaptive Square-Rooting Companding Technique for PAPR Reduction in OFDM Systems

Wisam F. Al-Azzo, and Borhanuddin Mohd. Ali

Abstract—This paper addresses the problem of peak-to-average power ratio (PAPR) in orthogonal frequency division multiplexing (OFDM) systems. It also introduces a new PAPR reduction technique based on adaptive square-rooting (SQRT) companding process. The SQRT process of the proposed technique changes the statistical characteristics of the OFDM output signals from Rayleigh distribution to Gaussian-like distribution. This change in statistical distribution results changes of both the peak and average power values of OFDM signals, and consequently reduces significantly the PAPR. For the 64QAM OFDM system using 512 subcarriers, up to 6 dB reduction in PAPR was achieved by square-rooting technique with fixed degradation in bit error rate (BER) equal to 3 dB. However, the PAPR is reduced at the expense of only -15 dB out-of-band spectral shoulder re-growth below the in-band signal level. The proposed adaptive SQRT technique is superior in terms of BER performance than the original, non-adaptive, square-rooting technique when the required reduction in PAPR is no more than 5 dB. Also, it provides fixed amount of PAPR reduction in which it is not available in the original SQRT technique.

Keywords—complementary cumulative distribution function (CCDF), OFDM, peak-to-average power ratio (PAPR), adaptive square-rooting PAPR reduction technique.

I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) modulation is chosen over a single-carrier solution in a broadband communications due to lower complexity of equalizers in high delay spread channels. The basic idea of OFDM transmission is to divide the available bandwidth into *N* sub-bands transmitted on different carriers. OFDM can be implemented efficiently by using fast Fourier transforms (IFFT and FFT) at the transmitter and receiver, respectively. Therefore, OFDM has been adopted in an ADSL modem for wired internet access and in many others non-portable wireless applications. Examples are: DAB, DVB, and WLANs [1], [2].

On the other hand, OFDM is characterized by a large envelope variation accompanied by high amplitude spikes compared to single-carrier systems using the same modulator format. This is attributed to the superposition process of the IFFT in the OFDM transmitter. The amplitude spikes make the OFDM signals very sensitive to distortion by nonlinear devices, such as analog-to-digital (A/D) converter, IFFT/FFT

processors with finite word length, and RF power amplifier [3]. To avoid distortion, the linear range of these devices should be wide enough. There are two main drawbacks in implementation and operation of devices with these specifications. The cost is the first issue, and the second important issue is the power efficiency of the power amplifier (PA) which is proportional inversely with extension of the amplification range. Therefore, a significant stream of research has focused on achieving low OFDM peak-to-average power ratio (PAPR) via various distortionless and distortion-inducing techniques [4], [5].

The PAPR reduction researches in OFDM systems using time domain for signal processing provided, in general, simple implementation solutions. The existing techniques process the complex-valued signals of the OFDM output symbol (block) when the signals are discrete and before the D/A converter. However, processing of signals in time domain causes distortion when it changes the envelope of the OFDM outputs. This effect happens in techniques such as clipping, clipping and filtering, and companding [6]-[8]. On the other hand, partial transmit sequences (PTS) techniques do not cause any distortion because it only changes the phase factors of the OFDM output signals [9], [10]. PTS techniques use iterative routine similar to the trial-and-error method for finding the optimum phase factors leading to lower PAPR. It is distortionless but is time-consuming, and needs large number of computations [11], [12].

This paper presents an adaptive square-rooting (SQRT) technique, a developed technique to our previous work, named square-rooting companding technique, given in [13] to reduce PAPR in OFDM systems. Here is also only the envelope of the complex OFDM signals is processed while the phases are kept unchanged. This process is similar to that given in [13], but it is applied adaptively whenever the PAPR is higher than a pre-defined threshold level. This is to enhance the BER performance of the original, non-adaptive, square-rooting companding technique. The effect of the SQRT companding process on the statistical transformation of OFDM signals, and consequently its impact on the PAPR reduction is analyzed.

This paper is organized as follows. Section II presents briefly the generation of OFDM signals, PAPR formulation, and the analysis of statistical properties of OFDM signals. Section III presents the proposed adaptive square-rooting technique for PAPR reduction in OFDM systems. Section IV and section V present the simulation results and conclusions, respectively.

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II. CONVENTIONAL OFDM SYSTEMS AND PAPR

In OFDM systems, a fixed number of successive input data samples are modulated first (e.g, PSK or QAM) as in the single-carrier systems, and then jointly correlated together using inverse fast Fourier transform (IFFT) at the transmitter side. IFFT used to produce orthogonal data subcarriers. Mathematically, IFFT combines all the input signals (superposition process) to produce each one of the output OFDM symbol $\{x_n\}_{n=0}^{N-1}$. The time domain complex OFDM signals is given as [1]

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi \frac{n}{N}k} , \ 0 \leq n \leq N-1 \qquad (1)$$

Where x_n is the n-th OFDM output signal, X_k is the k-th data modulated symbol in OFDM frequency domain, and N is the number of subcarriers.

However, OFDM systems are characterized by a large dynamic signals envelope due to the superposition process performed by the IFFT in the transmitter. PAPR is widely used to evaluate this variation of the output envelope. PAPR is an important factor in designing of both the high power amplifier and digital-to-analog converter, for generating error-free (or with minimum errors) OFDM output signals. The PAPR (in dB) is defined by the following equation [8]

$$PAPR = 10 Log_{10} \left\{ \frac{P_{Peak}}{P_{avg}} \right\} \qquad (2)$$

Where P_{peak} and P_{avg} are the peak and average power of output OFDM symbol, respectively, and they are computed as

$$P_{peak} = \max_{0 \leq n \leq N-1} |x_n|^2 \qquad (3)$$

and

$$P_{avg} = \frac{1}{N} \sum_{n=0}^{N-1} |x_n|^2 \qquad (4)$$

In the OFDM transmitter, the IFFT process changes the constellation shape of the processed signals. This change is attributed to the transformation in statistical properties of the real and imaginary parts, I and Q, of OFDM signals from uniform distribution at the frequency domain (before IFFT) into the Gaussian distribution at the time domain (after IFFT). Consequently, PAPR of the OFDM output signals is usually higher than that of the single-carrier systems using the same modulation format.

The complex-valued data modulated symbols X_k are considered as independent identically distributed (i.i.d.) random variables because the modulation is a memoryless process treating the data individually. In the OFDM systems, the summation of these i.i.d random variables is performed by the IFFT. Hence, according to the central limit theorem, IFFT changes the statistical distribution of the processed signals to complex Gaussian distribution [14]. Also, according to Parseval's theorem the IFFT operation does not alter the total average power of the processed signals [15]; therefore, the average power of the OFDM output symbol x_n

($P_{avg} = \sigma_{x_n}^2 = 2\sigma_{xI}^2$) is equal to the average power of the IFFT input data modulated vector X_k, $P_{avg} = \tilde{P}_{X_k} = N$ [6].

Based on statistical analysis of the complex Gaussian distribution [14], the distribution of the envelope (voltage) $R_n = \sqrt{x_{nI}^2 + x_{nQ}^2}$, $0 \leq n \leq N-1$ of the OFDM output signals x_n will follow Rayleigh distribution; while, the statistical distribution of the power of the same OFDM signals $\rho_n = x_{nI}^2 + x_{nQ}^2$, $0 \leq n \leq N-1$ follows central Chi-square distribution.

III. ADAPTIVE SQUARE-ROOTING PAPR REDUCTION TECHNIQUE

A. Original Square-Rooting PAPR reduction Technique

The block diagram of a typical OFDM system using the original SQRT technique for PAPR reduction is shown in Fig.1. By using the SQRT technique, the original OFDM output signals x_n is processed by (5) before they are converted into analog waveforms and amplified by the power amplifier

$$x'_n = \sqrt{|x_n|} \ e^{j\phi_n} , \quad 0 \leq n \leq N-1 \qquad (5)$$

x'_n is the new OFDM signal, and ϕ_n is the phase of x_n. During the entire signal processing, the phases of the OFDM output signals ϕ_n are kept unchanged while only the amplitudes are treated and changed [13].
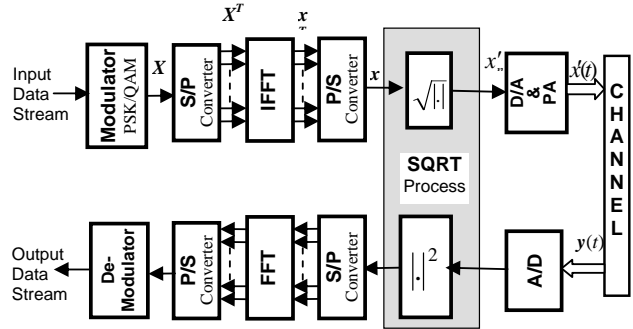

Fig. 1 Block diagram of an OFDM system using SQRT technique

For the complex Gaussian distributed signals, such as OFDM output signals, SQRT process changes the Rayleigh distribution of these signals into a Gaussian-like, close to Gaussian, distribution [13]; while the Chi-square distribution is converted, according to the analysis of these signals given in the previous section, to Rayleigh distribution. The later is because the Rayleigh distribution in such signals represents voltage, while the Chi-square distribution represents the power of the same signals. However, not only the statistical distribution is changed by the SQRT process, but the values of the mean μ and variance σ^2 of the processed OFDM output signals are also changed, and subsequently the values of the average power and peak power of these signals are altered

also.

To understand the effect of SQRT process on the power values of OFDM output signals, we assume normalized average power ($P_{avg} = 1$). When the average power is normalized, the value of the peak power is diminished by N because $P_{avg} = N$ for the same PAPR. This assumption is applicable for all OFDM symbols as the average power is constant and equal to N ($P_{avg} = N$). Hence, the PAPR can be analyzed according to (2) through the peak power only. The new value of normalized peak power is always greater than one because P_{Peak} is constantly greater than P_{avg} in all OFDM symbols. Therefore, the SQRT process always causes reduction in the value of peak power of the normalized OFDM symbols, and as a result the PAPR is reduced in all sizes of OFDM blocks, N.

In [13] the SQRT process is applied on the signals of all OFDM output symbols; therefore, the PAPR reduced without need to send side information. The SQRT process changes the distribution of the power signals to Rayleigh distribution and reduces the value of average power from N to $N^{1/2}$. The variance of Rayleigh distribution equals $(2 - \pi/2)\sigma^2$ [16] which is approximately equal to half the value of variance of the Gaussian distributed signals. The SQRT process in SQRT-OFDM system performs this statistical transformation, and therefore results in constant degradation in the BER rate equal to 3 dB because of decreasing of variance to the half of that of conventional OFDM system ($\sigma^2_{SQRT} \approx \frac{1}{2}\sigma^2_{Conv}$).

B. Adaptive SQRT-OFDM Technique

In this paper, we adopted adaptive application of the SQRT process on the OFDM output symbols. High PAPR values fortunately only take place with low probability in conventional OFDM systems (see results of table I); therefore, it is only necessary to perform PAPR reduction few times, when PAPR is high. This can be done by applying the SQRT process only on OFDM symbols those having PAPR greater than a pre-defined threshold level. The adaptive operation helps to restrain degradation in error performance due to the SQRT process. The adaptive SQRT technique for PAPR reduction uses the same SQRT-OFDM transceiver as given in Fig. 1, but the SQRT process should be applied as follows

$$x^a = \begin{cases} \sqrt{|x|}\angle\phi & for \quad PAPR > \gamma \\ x & for \quad PAPR \le \gamma \end{cases} \quad (6)$$

x^a is the adaptively square-rooted OFDM output symbol $x^a = \{x^a_n\}_{n=0}^{N-1}$, and γ represents the highest allowed PAPR threshold level. In the transmitter of the adaptive SQRT-OFDM system, the PAPR of each generated OFDM output signals should be tested first to decide the need of using the SQRT process.

Side information is needed at the receiver side to recover the original OFDM signals in the adaptive SQRT-OFDM system. Sending side information is needed only when the SQRT process is applied in the transmitter. A binary signal is enough for the side information; and therefore it can be sent on a separate channel or embedded inside the OFDM processed symbol. For simplicity, this work uses a separate channel to send side information.

IV. SIMULATION RESULTS

To show the PAPR reduction capacity, BER performance, and other features of the adaptive SQRT technique we considered an OFDM system using 512 subcarriers and a 64-QAM modulation scheme (*M*=64, *M* is the modulation order), simulated by randomly generated data. A complementary cumulative distribution function, CCDF=Prob{PAPR>PAPR$_o$}, was used to present the range of PAPR in term of a probability of occurrence. The simulations were ran for 100,000 OFDM symbols.

Fig. 2 shows the influence of the SQRT process on the statistical characteristics of the OFDM output signals. Only one OFDM block was used in this test. The figure shows clearly the change achieved in the statistical distribution for both the amplitude and power of the OFDM output signals resulting from the SQRT signal processing. A Rayleigh distribution of amplitude of the conventional OFDM output (Fig. 2.a) changed into a Gaussian-like distribution (Fig. 2.b); and at the same time, the Chi-square distribution of the OFDM output power signals (Fig. 2.c) changed to Rayleigh distribution (Fig. 2.d).

(a) Amplitude histogram of conventional OFDM

(b) Amplitude histogram of SQRT-OFDM

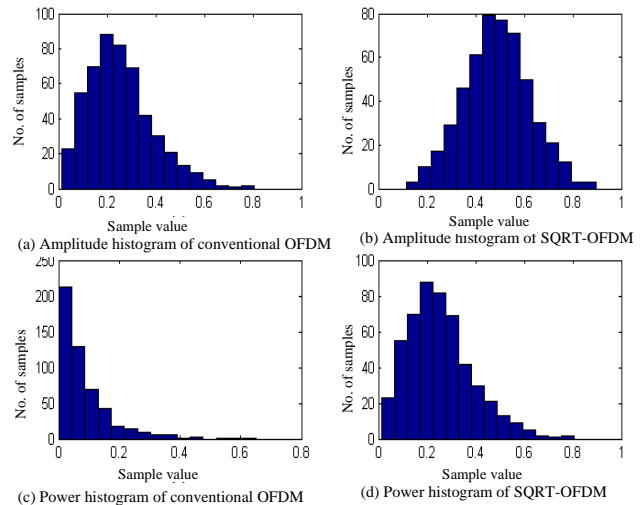(c) Power histogram of conventional OFDM

(d) Power histogram of SQRT-OFDM

Fig. 2 Impact of SQRT technique on statistical distribution of OFDM output signals

Fig. 3 shows the PAPR reduction capacity of the original, non-adaptive, SQRT-OFDM system. The figure shows the CCDF curves of OFDM signals before and after the application of the SQRT process. The PAPR measures at different probability of occurrence can be extracted from the CCDF curves for both the conventional OFDM and SQRT-OFDM systems. It can be noticed that the PAPR obtained by

the SQRT-OFDM system at CCDF=10^{-5} is reduced to about halve that obtained from the conventional OFDM system. The amount of reduction in PAPR due to the SQRT process is computed by

$$\Delta PAPR = PAPR - PAPR' \qquad (7)$$

Where $PAPR'$ is the PAPR of the SQRT-OFDM system. For this OFDM system, $PAPR = 13$ dB and $PAPR' = 7$ dB leading to 6 dB reduction in PAPR, $\Delta PAPR = 6$ dB.
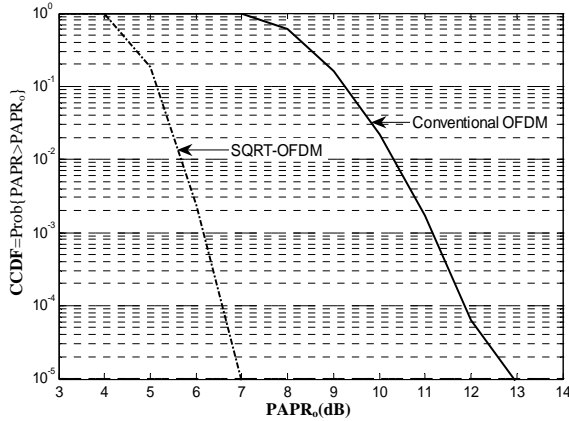

Fig. 3 PAPR reduction performance of SQRT technique

Table I presents the probability of occurrence of OFDM output symbols, represented by CCDF, of some relatively high PAPR values of the conventional 64QAM-OFDM system shown in Fig. 3. The PAPR-CCDF results of table I indicate that there are only small numbers of OFDM symbols where PAPR needs to be reduced. In general, this conclusion is true when the required amount of reduction in the PAPR does not exceed 3 dB.

TABLE I
PROBABILITY OF OCCURRENCE FOR DIFFERENT PAPR VALUES
IN THE CONVENTIONAL 64QAM-OFDM SYSTEM

PAPR (dB)	13	12	11	10	9	8
CCDF	10^{-5}	6×10^{-5}	2×10^{-3}	2×10^{-2}	2×10^{-1}	6×10^{-1}

Fig. 4 shows the PAPR reduction capacity of the proposed adaptive SQRT-OFDM system. The figure presents the CCDF curves of different PAPR threshold levels. Here, the adaptive SQRT technique offered, in contrast to the original SQRT technique, controlled PAPR reduction performance. This can be noticed from the CCDF curves in the figure, where the maximum PAPR obtained for all tests are close to their pre-defined threshold level γ. For example, 9 dB for $\gamma = 10$ dB, 8 dB for $\gamma = 9$ dB, and 7 dB for $\gamma = 8$ dB.
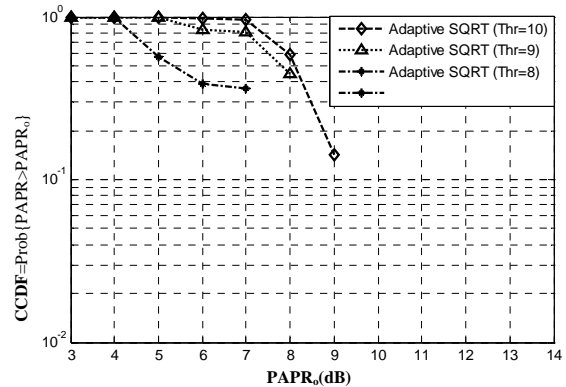

Fig. 4 PAPR reduction performance of the adaptive SQRT technique

Fig. 5 presents the BER performance over an AWGN channel for the conventional OFDM, original SQRT-OFDM, and the adaptive SQRT-OFDM systems. The adaptive SQRT-OFDM system uses three different values of PAPR threshold: $\gamma = 10$ dB, $\gamma = 9$ dB, and $\gamma = 8$ dB, to test the error performance at various PAPR reduction capacities. Complex forms of AWGN noise signals were generated by MATLAB for different E_b/N_o ratios and added to the OFDM signals of the three aforementioned OFDM systems to perform this test. The results of Fig. 5 show, in general, improvement in the error performance due to the use of adaptive mode of SQRT technique compared to the original SQRT technique. This can be observed in the figure when we compare the difference in E_b/N_o ratios obtained from the original and adaptive SQRT techniques at particular BER level, e.g. BER= 10^{-4}, with that obtained from the conventional OFDM system.
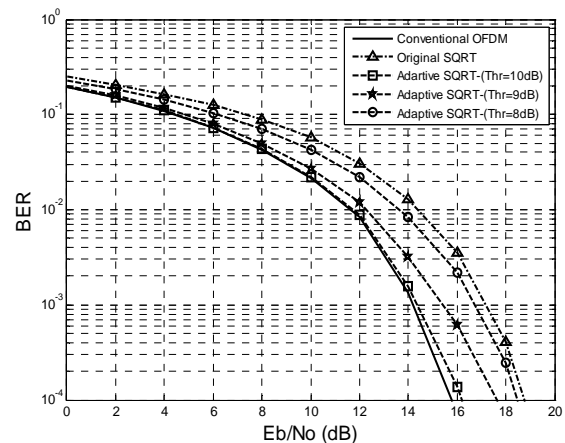

Fig. 5 BER performance of the adaptive SQRT technique

More specifically, three different profiles can be recognized in Fig. 5. First, when the required amount of PAPR reduction is relatively small, as in $\gamma \leq 10$ dB, the adaptive SQRT technique offered reduction in PAPR with about no degradation in BER performance. This result can be seen in the figure from the closeness of BER performance obtained at $\gamma = 10$ dB to that obtained from the conventional OFDM system. Second, when 8 dB $< \gamma <$ 10 dB, the adaptive SQRT

technique offered better performance than the original SQRT technique as the later offers always 3 dB degradation in the BER performance. Third, when $\gamma \geq 8$ dB, the adaptive operation of SQRT technique did not offer improvement in the BER performance compared to the original SQRT technique. The last result can be attributed to increased percentage of the number of the processed OFDM symbols by the SQRT operation. Back to table I, this happens when CCDF, the probability of occurrence, is greater than 6×10^{-1} (when $\gamma \geq 8$).

The impact of the SQRT process on the power spectral density of the OFDM output signals is depicted in Fig. 6. Oversampling was used to inspect the out-of-band radiation caused by the amplitude distortion due to the SQRT process. The figure shows the power spectral density of one block of the SQRT-OFDM signals using N=512 subcarriers. The oversampling factor was set to J=4. Fig. 6 shows that the level of the out-of-band shoulder is about -15 dB lower than that of the in-band signals. However, other tests by using other systems parameters (N=256 and M=16, N=128 and M=32, and N=64 and M=32) showed that the maximum level of the shoulders are always limited to this level, -15 dB. The power level of out-of-band radiation is the same in both the original and adaptive SQRT techniques. On the other hand, the difference comes from that there is always out-of-band radiation accompanied to all OFDM output symbols in the original SQRT technique because all of these symbols are processed; while in the adaptive SQRT technique the out-of-band radiation occurred only with the processed OFDM symbols.
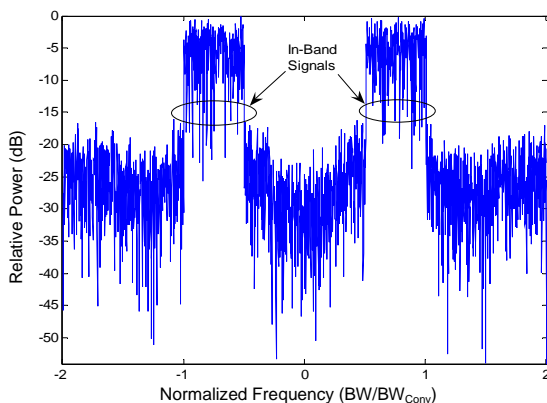

Fig. 6 Power spectral density of SQRT-OFDM signals

V. CONCLUSIONS

Initially, we proposed PAPR reduction technique using adaptive square-rooting companding process on the OFDM output signals. This process changes the statistical characteristics of the treated OFDM output signals from Rayleigh to Gaussian-like distribution. At the same time the values of power signals, average and peak, diminished and as a result the PAPR is reduced. The original square-rooting technique can reduce the PAPR up to 6 dB at CCDF=10^{-5}.

Adaptive square-rooting technique offers better error performance than the original square-rooting technique when the required reduction in PAPR is no more than 5 dB. Also, it provides control to the reduction amount of PAPR, where this facility is not available in the original square-rooting technique.

Adaptive square-rooting technique is similar to the original square-rooting technique reduces PAPR at the expense of some creation of out-of-band radiation. The shoulder level of the out-of-band radiation was shown to be always less than -15 dB to in-band signals. Finally, this technique is more suitable for OFDM applications those do not have sophisticated processing facility, since it allows considerable reduction in PAPR with low computational complexity and either no or only slight degradation in the error performance.

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