

# Rarefactive and Compressive Solitons in Warm Dusty Plasma with Electrons and Nonthermal Ions

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**Abstract**—Dust acoustic solitary waves are studied in warm dusty plasma containing negatively charged dusts, nonthermal ions and Boltzmann distributed electrons. Sagdeev pseudopotential method is used in order to investigate solitary wave solutions in the plasmas. The existence of compressive and rarefactive solitons is studied.

**Keywords**—Nonthermal, Soliton, Dust, Sagdeev potential

## I. INTRODUCTION

IN recent years, there have been considerable interests in understanding the different types of collective processes in plasmas containing electrons, ions and charged micron-sized grain particles. Such plasmas occur frequently in many astrophysical systems including the interplanetary medium, planetary rings, asteroids, cometary tails, interstellar clouds, nebulae, aurora etc. and they are also produced in plasma discharges, optical fibres, dusty crystals, semiconductors as well as regions of hot fusion plasma and in devices for plasma-assisted material processing [1–7]. For low frequency modes, the grain dust can be described as negative ions with large mass and large charge. In particular it has been shown that dusty plasmas with inertial dust fluid and Boltzmann distributed ions admit only negative solitary potentials associated with nonlinear dust acoustic wave [8]. They ignored dust temperature which may not be negligible [9]. If one ignores the dust charge fluctuation dynamics, the dusty plasma can be regarded as a multicomponent plasma with several ionic species [10, 11]. However, the existence of a solitary acoustic wave in a multicomponent plasma had been reported by Dwivedi [12], contrasting the observation on a dust acoustic (DA) wave by Rao et al. [4], where the dust-particle mass provides the inertia and the pressures of inertialess electrons and ions provide the restoring force. Relevant observations were also made by other authors mentioned above [10, 11]. On another side, space plasma observations indicate clearly the presence of ion populations which are far away from their thermodynamic equilibrium [13–15]. The Vela [16] satellite observed non-thermal ions from earth's bow-shock, *Phobos 2* [17] satellite observed the loss of energetic ions from the upper ionosphere of the Mars and *Nozami* [18] satellite observed very large velocity protons near the earth in the vicinity of the moon. Also Lundlin *et al* [17] have shown that for the planet having in not so strong magnetic field, the solar wind impacting with the planetary

atmosphere results in nonthermal ion flux. Recently, motivated by the latter class of events, Cairns et al. [19] used non-thermal distribution of electrons in order to studying the ion acoustic solitary structures observed by the FREJA satellite. The presence of nonthermal ion distribution with cold dust, leads to the possibility of co-existence of large amplitude compressive as well as rarefactive dust acoustic solitary waves [20–22]. Most of the studies in multi-species plasmas have focused on deriving Korteweg-de Vries (KdV) and Kadomstev-Petviashvili (KP) equations using reductive perturbation technique [23,24]. Sagdeev's pseudo-potential technique is used to investigate the existence of double layers in dusty plasmas with non-thermal electrons and two temperature ions in [25]. M. Tribeche et al. [26] investigated the effect of nonthermal electrons with excess of fast energetic electrons on large amplitude electrostatic solitary waves in charge varying dusty plasmas. Ghosh et al. [27] have investigated the linear dust acoustic wave propagation characteristics incorporating the effects of nonthermal ions, the isothermal pressure variations of dust grains and both nonadiabatic and adiabatic dust charge variations. The effects of low dust charging rate compared to the dust oscillation frequency and nonthermal ions on dust acoustic wave have been investigated in [28]. Dust acoustic double layers in a four component dusty plasma have been studied by Mandal et al. [29]. Our aim is to study the trace the influence of relevant physical parameters on these, by considering a dusty plasma which consists of nonthermal ions, thermal electrons and warm dust grains. We shall also examine the possibility of the formation of localized solitonic solutions and discuss their characteristics. Among other physical parameters discussed, our formulation leaves open the choice of the dust pressure ("temperature") scaling [30–32]. Another important effect studied here is the influence of nonthermal ions [33–37] in our system. In the present letter we study large amplitude solitary waves not only with finite dust temperature but also incorporating the effect of nonthermal ion distribution. We can and stress that the results of the present investigation should lead to laboratory experiments which deal with the demonstration of DAW waves in a warm dusty plasma with negative grains and non-thermal ions. In the present paper we study large amplitude solitary waves not only with finite dust temperature but also incorporating the effects of nonthermal ion distribution. Sagdeev pseudopotential method [38], which takes complete nonlinearity into consideration, is used in order to set up energy integral to characterizing the dust acoustic solitary wave.

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II. BASIC FORMULATION

We consider unmagnetized, collisionless warm dusty plasma comprising of nonthermal ions, isothermal electrons and negatively charged dust. Dimensional equations governing the dynamics of the dusty plasma are [39,42]

$$\frac{\partial n_d}{\partial t} + \frac{\partial(n_d u_d)}{\partial x} = 0$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} + \frac{\sigma}{n} \frac{\partial P}{\partial x} = \frac{\partial \phi}{\partial x} \tag{1}$$

$$\frac{\partial P}{\partial t} + u_d \frac{\partial P}{\partial x} + 3P \frac{\partial u_d}{\partial x} = 0$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_d + n_e - n_i$$

we have taken the equation of state as

$$P = n_d^\gamma P_0 \tag{2}$$

Here  $\gamma$  is the ratio of specific heats and taken as  $\gamma = 3$ ,

$$\sigma = \frac{T_d}{T_{eff}} \text{ where } T_d \text{ is the dust temperature and}$$

$$T_{eff} = \left[ n_{0d} Z_{0d} / \left( \frac{n_{0e}}{T_e} + \frac{n_{0i}}{T_i} \right) \right]$$

is the effective temperature.

$Z_{0d}$  is the unperturbed number of charges on the dust particles.  $n_d$  and  $u_d$  are the dust particle density and dust particle velocity and they are normalized by  $n_{0d}$  (unperturbed

dust density) and  $c_d = \sqrt{\frac{T_i}{m_d}}$  (dust acoustic speed),

respectively.  $\phi$  is the electrostatic wave potential normalized by  $\frac{T_i}{e}$ , where  $T_i$  is the ion temperature. The time and space

variables are in the units of  $\omega_{pd}^{-1} = \sqrt{m_d / 4\pi n_{d0} Z_{0d}^2 e^2}$  the dust plasma period and the Debye length  $\lambda_d = \sqrt{T_{eff} / 4\pi Z_{0d} n_{d0} e^2}$ , respectively [9].  $n_e$  and  $n_i$  are electrons and ions number density, respectively. Charge neutrality at equilibrium requires that

$$Z_{0d} n_{0d} + n_{0e} = n_{0i} \tag{3}$$

where  $n_{0e}$ ,  $n_{0i}$  and  $n_{0d}$  are the unperturbed electrons, ions and dust number densities, respectively. To study the effect of a nonthermal ion distribution on dust acoustic wave, we choose a more general class of ion distribution [40] which includes the population of nonthermal ions.

$$n_i = \frac{1}{1-\mu} (1 + \beta\phi + \beta\phi^2) e^{-\phi} \tag{4}$$

where  $\mu = n_{0e} / n_{0i}$  and  $\beta = \frac{4\alpha}{1+3\alpha}$  which  $\alpha$  is nonthermal

parameter and indicates the population of fast ion [41] and is used in our calculations. And for electrons with Boltzmann distribution

$$n_e = \frac{\mu}{1-\mu} e^{\delta\phi} \tag{5}$$

where  $\delta = \frac{T_i}{T_e}$ , where  $T_e$  is the electron temperature.

In order to investigate the properties of large amplitude stationary dust acoustic solitons, we assume that all the dependent variables in nonlinear equations (1-4) depend only on a single variable  $\xi = x - Mt$  where  $M$  being the soliton velocity normalized by  $c_d$ . Equations (1) in the stationary frame can be integrated to give

$$n_d = \frac{\sqrt{2M}}{\sqrt{M^2 + 2\phi + 3\sigma + \sqrt{(M^2 + 2\phi + 3\sigma)^2 - 12\sigma M^2}}} \tag{6}$$

where we have used boundary conditions for localized disturbance, viz,  $\phi \rightarrow 0$ ,  $u_d \rightarrow 0$  and  $n_d \rightarrow 1$  as  $|\xi| \rightarrow \infty$ .

Substituting  $n_d$  from (6) in Poisson equation and following Sagdeev's pseudopotential method along with appropriate boundary conditions, we obtain

$$\frac{1}{2} \left( \frac{d\phi}{d\xi} \right)^2 + V(\phi) = 0 \tag{7}$$

where

$$V(\phi) = \left[ \frac{1}{1-\mu} \left( 1 + 3\mu + \frac{\mu}{\delta} \right) + M^2 + \sigma \right] - \frac{1}{1-\mu} \left[ 1 + 3\beta + \beta\phi^2 + 3\beta\phi \right] e^{-\phi} - \frac{1}{1-\mu} \left( \frac{\mu}{\delta} \right) e^{\delta\phi}$$

$$- \frac{\sqrt{2}}{2} M \left\{ \frac{\sqrt{M^2 + 2\phi + 3\sigma + \sqrt{(M^2 + 2\phi + 3\sigma)^2 - 12\sigma M^2}} + 4\sigma M^2}{\left[ M^2 + 2\phi + 3\sigma + \sqrt{(M^2 + 2\phi + 3\sigma)^2 - 12\sigma M^2} \right]^3} \right\} \tag{8}$$

This result is consistent with the results which have been reported in [42] and [43] for unmagnetized and magnetized cases in warm dusty plasmas without electrons, respectively. The above results also are in a good agreement with [21,22] for dusty plasmas with cold dust grains. Equation (7) can be regarded as an 'energy integral' of an oscillating particle of

unit mass with a velocity  $d\phi/d\xi$  and position  $\phi$  in a potential  $V(\phi)$ . Further it is clear that  $V(\phi)=0$  and  $dV(\phi)/d\phi=0$  at  $\phi=0$ . Solitary wave solution for Eq.(8) exists if  $d^2V/d\phi^2 \leq 0$  at  $\phi=0$ , so that the zero as a fixed point is unstable. All the specified conditions are satisfied. Besides that  $V(\phi)$  should be negative between  $\phi=0$  and  $\phi_m$  where  $\phi_m$  is some maximum or minimum potential for compressive or refractive solitons respectively.

### III. DISCUSSION

It is obvious that  $V(\phi)$  is a very sensitive function of plasma parameters ( $\sigma, \alpha, \mu, \delta$  and  $M$ ). Since dust temperature and nonthermal ion are the main considered parameters, it is useful to investigate the dynamics of solitons as functions of these parameters. Figures 1 and 2 present  $V(\phi)$  as functions of  $\phi$  for different values of  $\sigma$  with  $\alpha=0$  (Fig. 1) and  $\alpha=0.55$  (Fig. 2). It can be seen that both rarefactive and compressive solitons are created. These figures also show that increasing the dust temperature significantly affected on both the compressive and the rarefactive solitons. These figures also show that for a range of values of  $\sigma$ , solitary waves will not appeared. Figures 3 and 4 show  $V(\phi)$  as a function of  $\phi$  for different values of  $\alpha$ . These figures show the comparison of the Sagdeev pseudo-potential profiles for different values of  $\alpha$  by choosing the values of  $\sigma=0.001, 0.1$ , respectively. Figure 3 show that for very low dust temperature, rarefactive solitons exist over a thicker range of  $\alpha$  values. It is seen that compressive and rarefactive solitons can be appeared in the plasma. There is a shift in the value of  $V(\phi)$  as well as  $\phi_m$  as  $\alpha$  is decreased. Further investigations show in the existence of nonthermal ions compressive solitons appear at first. Then compressive solitons immediately disappear when we start increasing the value of  $\alpha$ . However, rarefactive solitons persist till the specific value of  $\alpha$ . It is clear that it is possible to investigate the nonlinear wave structures over a wider range of parameters space. It can be also show that with the increase in the dust temperature, compressive solitons change to rarefactive solitons and a further increase in the dust temperature leads to the appearance of compressive solitons. It should be mentioned that for cold dust fluid ( $\sigma=0$ ) and for Maxwellian ion distribution ( $\alpha=0$ ), our results completely agree with those obtained by Mamun et al. [8]. Our findings also is consistent with the one made by Verheest et al. [21] for  $\sigma=0$  and also by the Das et al. [44] for  $\sigma \neq 0, \alpha \neq 0$ . It should be noted that in this work the chosen method to study the effects of dust temperature and nonthermal ions on the dust acoustic solitary waves is different from the other methods taken by the others and our findings are in good agreement with their results. The limitation of the present analysis is on rarefactive and compressive solitary waves as

separately. It can be investigated that compressive and refractive solitons coexist for specific values of  $\alpha$  and  $\sigma$ . In these cases, it might add a new scenario to the soliton dynamics to determine the border between two kinds of solitons. This work can be also extended in a dusty plasma with positive dust charge fluctuation in the presence of non-thermal electrons or vortex like ions or vortex like electrons. It may be pointed out that the results of this study should also be important in understanding some nonlinear behavior of electrostatic waves in Saturn's rings and Halley's comet [45,46] and astrophysical dusty plasma systems [47].

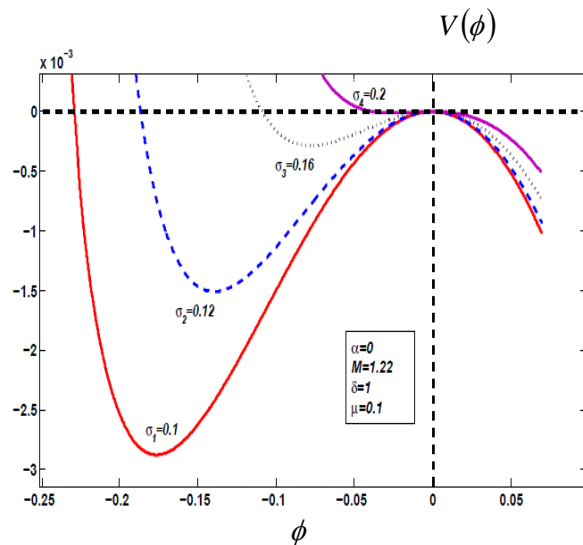


Fig. 1 The Sagdeev potential  $V(\phi)$  with respect to  $\phi$  for fixed value of  $M, \alpha=0, \mu, \delta$  and different values of  $\sigma$

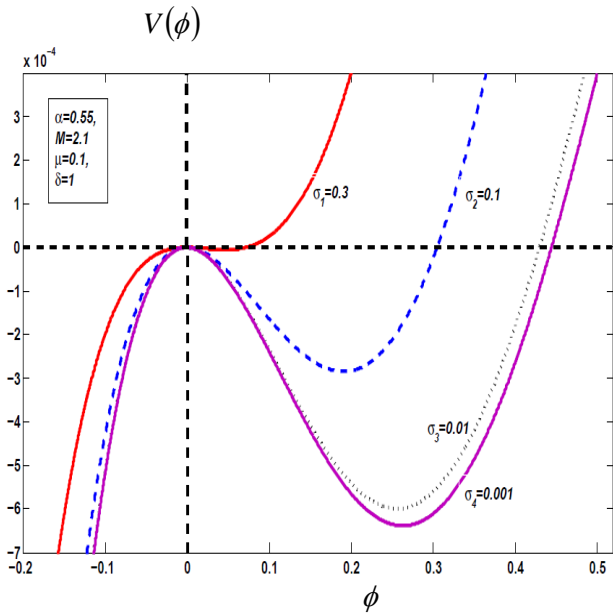


Fig. 2 The Sagdeev potential  $V(\phi)$  with respect to  $\phi$  for fixed value of  $M, \alpha=0.55, \mu, \delta$  and different values of  $\sigma$

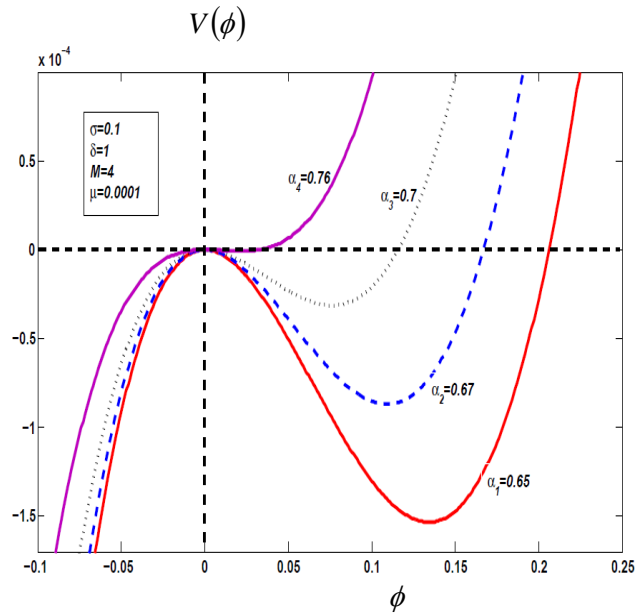


Fig. 4 The Sagdeev potential  $V(\phi)$  with respect to  $\phi$  for fixed value of  $M, \sigma=0.1, \mu, \delta$  and different values of  $\alpha$

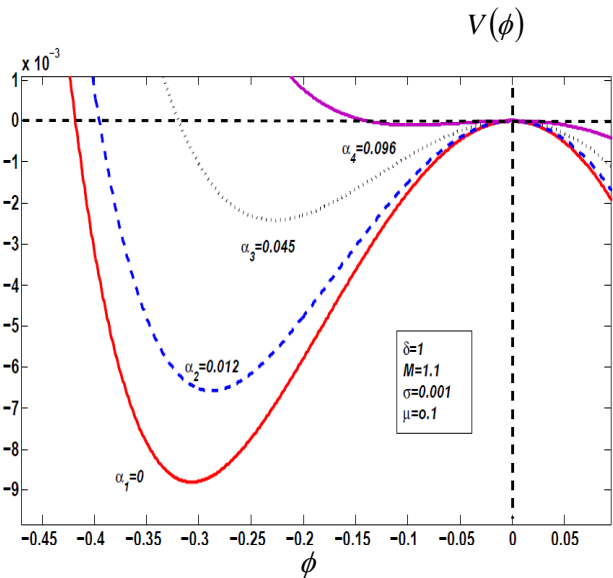


Fig. 3 The Sagdeev potential  $V(\phi)$  with respect to  $\phi$  for fixed value of  $M, \sigma=0.001, \mu, \delta$  and different values of  $\alpha$

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