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A Dual Model for Efficiency Evaluation Considering Time Lag Effect

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***Abstract*—**A DEA model can generally evaluate the performance using multiple inputs and outputs for the same period. However, it is hard to avoid the production lead time phenomenon some times, such as long-term project or marketing activity. A couple of models have been suggested to capture this time lag issue in the context of DEA. This paper develops a dual-MPO model to deal with time lag effect in evaluating efficiency. A numerical example is also given to show that the proposed model can be used to get efficiency and reference set of inefficient DMUs and to obtain projected target value of input attributes for inefficient DMUs to be efficient.

***Keywords*—**DEA, efficiency, time lag, dual problem.

I. Introduction

DEA evaluates the relative performances of comparable units having the authority to make decisions. DEA models use multiple input and output factors of DMUs in efficiency assignments. The efficiency of a DMU is defined as the ratio of weighted outputs to weighted inputs. Since the first Data Envelopment Analysis (DEA) model was developed by Charnes, Cooper and Rhodes (CCR) [1], there are various extensions of the original CCR model [2]-[6]. In special, Banker, Charnes, and Cooper (BCC) extended the CCR model to accommodate technologies that exhibit variable returns to scale [2]. There are many extended models of the basic DEA model. A Generalized DEA model (GDEA) was developed by Yu, Wei and Brockett [7]. Post and Spronk proposed an interactive DEA procedure (IDEA) and introduced a combined DEA model that solves a single linear program to compute the efficiency values of all DMUs [8].

DEA models generally assume that inputs are converted to outputs in the same period. However, this assumption may not be appropriate to some situations such as the performance evaluation of R&D activity and educational activity. Consider academic papers published or patents applied or registered as an output of a research activity. A published journal paper may result from research efforts over several years because the review process of an article for a journal could take some time. That is, production lead time is required to produce the output, such as journal papers or patents, from starting a research project. Furthermore, this production lead time varies according to research areas. In other words, there is a certain length of time lag between the consumption of inputs and the production of outputs. The length of time lag must be dependent on research area which is considered to obtain efficiency of research activity.

In this case, general DEA models cannot be applied to obtain efficiency value of DMUs because inputs in a given period may contribute to outputs of several subsequent periods. Ozgur Ozpeynirci and Murat Kokslan [9] developed the Multi-period Input (MpI) model to capture the time lag between the inputs and outputs in DEA. This model was developed under assumption that the output of a period is produced by consuming the inputs of one or more previous periods from the output period. They added a lag period index t to Post and Spronk's model [8] in order to take account for time lag effect. This model is to find the efficiency values of all DMUs for all periods by a single model. Zhang and Jeong [10] suggested a DEA model which interprets the time lag effect in other view. The model was developed under the view that the input of a certain period can contribute to the outputs of one or more subsequent periods from the input period. That is, the outputs of one or more subsequent periods are thought to be produced by consuming the input of a period. However, it is not possible to explain the relation with the consumption periods of inputs and the production periods of outputs. The paper suggested a multi-period output model (MPO) model to capture another interpretation of time lag effect.

This paper suggests a dual model of MPO model for calculating efficiency considering time lag effect. The dual model can give a reference set of inefficient DMUs and can be used to obtain input excess and output shortfall of an inefficient DMU. Section II gives a brief explanation about the difference of MPI and MPO model. Section III suggests a dual model of MPO model. A numerical example is given in Section IV.

II. Multi-Period Output Model

For efficiency evaluation of long-term projects, the effect of time lag due to production lead time should be captured to obtain more exact efficiency of research activities. Most of DEA models didn't take consideration of the time lag effect in defining efficiency. At first, Ozpeynirci and Kokslan suggested the MPI DEA model to capture time lag effect that there might be between the consumption of inputs and the production of outputs. Zhang & Jeong [10] dealt with the time lag effect in other view and developed Multi-period output model to consider the time lag effect in efficiency evaluation. The model assumed that the input of a period could contribute to not only the outputs of the same period but also the outputs of subsequent periods from the period.

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A. Time Span for Time Lag Models

The difference in the definition of efficiency should be clarified between MPI and MPO models. To do this, it is required to compare the time span for efficiency measure used by models. Table I compares the time span to be used to calculate efficiency of a DMU by MPI and MPO model. Assume that the longest time lag for producing research output is 2 periods, i.e., PM = 2. In the MPI model suggested by Ozpeynirci and Kokslan [9], they thought that the outputs of a period are produced by the consumption of the inputs of one or more previous periods of the period. The idea of MPI model is to measure the efficiency value of a DMU using the ratio of its weighted output for the third period to the weighted input for 1-3 periods. That is, the underlying assumption of the efficiency definition is that the inputs of a DMU for some multiple periods contribute to the outputs of a single period, which is the last period of the multiple periods.

However, MPO model approached to deal with time lag effect in other view. That is, for a DMU, the inputs of a period can be interpreted to contribute to the outputs not only of the same period but also one or more subsequent periods of the period. In the publication process of research papers, we often find that the consumption of inputs of a year could produce the outputs in the same year or in one or more years later. The MPO model was based on this interpretation of time lag effect. For the first period, the efficiency value of a DMU is defined by the ratio of its weighted output for periods 1-3 to its weighted inputs in the first period in the case of MPO model.

TABLE I
TIME SPAN FOR DEFINING EFFICIENCY OF MPI AND MPO MODEL

Model	T	1	2	3	4	5	6	7	8
MPI	Input								
	Output								
MPO	Input								
	Output								

As we can see in the Table I the definition of efficiency measure in MPO model is different from that of MPI model. Because of this difference, the periods for which we can get the efficiency values of DMUs depend on the model to be applied for considering time lag. For the same data set, we can obtain efficiency values for periods 1-6 using MPO model while MPI model can give efficiency values for periods 3-8.

B. Multi-Periods Output (MPO) Model

The MPO model assumed that the inputs of a period contribute to the outputs not only of the same period but also subsequent PM periods of the period. Let h_{jt} be the efficiency of the input used in the t^{th} period by DMU j. Let x_{ijt} denote the amount of the i^{th} input consumed by DMU j in the period t. Let v_{ijt} be the weight of the i^{th} input consumed by DMU j in the period t. Let $Y_{rj(t+p)}$ represent the amount of the r^{th} output produced by DMU j in the period t + p. Let u^p_{rjt} denote the weight of the r^{th} output produced by DMU j p periods after from the period t. Since there is a time lag of PM periods, the outputs corresponding to the input of the 1^{st} period are from the first period to the $(PM+1)^{th}$ period. The following MPO model finds the efficiency scores for periods [1, T-PM]. When PM = 0, (MPO) is equivalent to the combined DEA model.

(MPO);

$$\text{Max } \sum_{p=0}^{PM}\sum_{r=1}^{s} u^{p}_{rj_0t_0} Y_{rj_0(t_0+p)} \tag{1}$$

subject to

$$\sum_{i=1}^{m} v_{ij_0} X_{ij_0t_0} = 1 - \sum_{i=1}^{m} v_{ij_0} X_{ikt_0} + \sum_{p=0}^{PM}\sum_{r=1}^{s} u^{p}_{rj_0t_0} Y_{rk(t_0+p)} \le 0, k = 1, \dots, n;$$

$$u^{p}_{rj_0t_0}, v_{ij_0} \ge 0 \tag{2}$$

III. DUAL PROBLEM OF MPO MODEL

Let's consider the dual problem of (MPO) with the dual variables θ and $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_n)$ corresponding (1) and (2), respectively. The dual problem is as follows.

(Dual-MPO)

$$\min \quad \theta$$

$$s.t. \quad \sum_{j=1}^{n} \lambda_j X_{ijt_0} \le \theta X_{ij_0t_0}, i = 1, \dots, m$$

$$\sum_{j=1}^{n} \lambda_j Y_{rj(t_0+p)} \ge Y_{rj_0(t_0+p)},$$

$$r = 1, \dots, s;\ p = 0, \dots, PM$$

$$\lambda_j \ge 0,\ j = 1, \dots n,\ \ \theta \text{ is free} \tag{3}$$

By the strong duality relation, (Dual-MPO) gives the same objective function value as (MPO) when (MPO) has an optimal solution. (Dual-MPO) always has a feasible solution $\theta = 1, \lambda_{j_0} = 1, \lambda_i = 0\ (i \ne j_0)$ and $\theta \ge 0$ by (3). Hence the optimal θ, denoted by θ^*, should be $0 \le \theta^* \le 1$. This follows the strong duality relation and we can get the efficiency of DMU j_0 by solving (Dual –MPO).

DMUs for which $\theta^* < 1$ are inefficient and DMUs for which $\theta^* = 1$ are boundary points of the production possibility set. Some boundary points may be in efficient when they have nonzero slacks. So, we can't check the efficiency of a DMU with θ^*. But, we can avoid such cases by solving the following LP problem [11].

(SP)

$$\max \quad \sum_{i=1}^{m} s_i^- + \sum_{p=0}^{PM}\sum_{r=1}^{s} s_{r(t_0+p)}^+$$

$$s.t. \quad s_i^- = \theta^* X_{ij_0t_0} - \sum_{j=1}^{n} \lambda_j X_{ijt_0}, i = 1,...,m;$$

$$s_{r(t_0+p)}^+ = \sum_{j=1}^{n} \lambda_j Y_{rj(t_0+p)} - Y_{rj_0(t_0+p)},$$

$$r = 1,...,s;\ p = 0,...,PM$$

$$\lambda_j \geq 0,\ j = 1,...n$$

$$s_i^- \geq 0, i = 1,...,m;$$

$$s_{r(t_0+p)}^+ \geq 0, r = 1,...,s;\ p = 0,...,PM; \tag{4}$$

In (SP), there are two kinds of slacks, s_i^- are input excesses and $s_{r(t_0+p)}^+$ are output shortfalls. Note that any choice of slacks does not affect the optimal θ^* obtained by (Dual-MPO). We can get the possible input excesses and output shortfalls by solving (SP). A DMU is efficient if and only if both $\theta^* = 1$ and all slacks are 0 [11]. When DMU j_0 is inefficient, we can get the target value ($\hat{x}_{j_0}, \hat{y}_{j_0}$) as follows with the optimal slacks to (SP) [11].

$$\hat{x}_{j_0} = \theta^* x_{j_0} - s^{-*} = \sum_j x_j \lambda_j^* \leq x_{j_0}$$

$$\hat{y}_{j_0(t_0+p)} = y_{j_0(t_0+p)} + s_{(t_0+p)}^{+*} = \sum_j y_{j(t_0+p)} \lambda_j^* \geq y_{j_0(t_0+p)} \tag{5}$$

$x_j \in R^m$ and $y_j \in R^{R\times P}$ are the vectors of input and output values for each DMU and $s^{-*} \in R^m$ and $s_{(t_0+p)}^{+*} \in R^m$ are the optimal slack vectors obtained from (SP).

In (SP), the objective function is to maximize the sum of slacks but we can consider another weighted sum of the slacks. In other words, we can consider another function $\sum_{i=1}^{m} w_i^- s_i^- + \sum_{p=0}^{PM}\sum_{r=1}^{s} w_{r(t_0+p)}^+ s_{r(t_0+p)}^+$ using the nonnegative weights $w_i^-, w_{rp}^+ \in R^+$ instead of the objective function of (SP). (SP) is the case all weights for slacks are 1. With the objective function, we can control the importance of input and output factors by using the different weight factors. Suppose that we cannot control some input factors then it is impossible to change the input factor to the target value. In this case, we can set the weight value for those uncontrollable input factors are 0 and all the other weights are 1. Then, the optimal solution gives a solution to maximize the sum of slacks for controllable input factors. The results also give another target values adjusting controllable input factors as much as possible.

IV. Numerical Example

The 21st century frontier R&D program is a long-term national R&D program of Korea, for selective and intensive development of strategic technologies to enhance national scientific competitiveness to the level of advanced countries by 2010. The objective of the program is to develop the national economy through improving national competitiveness, public welfare, quality of life to the level of advanced countries and creating new industries through the development of future technologies. Currently, 24 research centers are being supported by this program since two project centers were launched in 1999. Each center is granted about 9 million US dollar per year by the government and its running period is no more than ten years from its start.

The number of subprojects supported by these 24 centers is totally 2,670 subprojects. However, the long term projects are required to apply the MPO model considering time lag effect. Thus, the data about input factors and output factors of the research center SG09 among these 24 centers will be used to analyze the efficiency of research activity. There are 17 subprojects having been supported for ten years by the SG09 center. There are four input variables, research expenditure (million Korean Won), the number of researchers (Ph.D, MS, BS). The number of published papers (SCI journal and non SCI journal) and the number of applied and registered patents are considered as outputs of research activity. These input and output data for 17 DMUs are used to find efficiency value of period 8 using the dual model of MPO model.

TABLE II
Efficiency Values and λ Values of Inefficient DMUS in Center SG09

DMU	θ	Reference set	λ
C	0.67230	{A,D,F,N}	{0.1892, 0.1757, 0.4865, 0.0270}
E	0.80625	{D,Q}	{1.2188, 0.1250}
H	0.79974	{A,D,I,M}	{0.2823, 0.1523, 1.1425, 0.0210}
K	0.81818	{A,D}	{0.4545, 0.2727}
L	0.94583	{A,G,I}	{0.1667, 0.3333, 0.4167}
O	0.28797	{A,D,G}	{0.6519,0.2278,0.2342}
P	0.70294	{A,D,I,Q}	{0.3725,0.2549,0.1066667E -05,0.2353}

Table II shows the results of MPO model using the data set of 17 projects supported for 10 years by research center SG09. The table gives efficiency values, reference sets and λ values of inefficient DMUs. Table III shows the target values of research grant for each inefficient DMU to become an efficient DMU. These target values were obtained by solving the model (SP) for each inefficient DMU. In this model, only s_1^- for an input excess in objective function of (SP) was used because only research grant among four input factors is controllable Research center SG09.

TABLE III
PROJECTED RESEARCH GRANT FOR EACH INEFFICIENT DMU TO BE EFFICIENT

DMU	Research grant (m₩)	Projected grant (m₩)
C	200	134.459
E	250	201.563
H	150	119.961
K	150	122.727
L	100	94.583
O	1000	186.646
P	200	140.588

V. CONCLUSION

Basic DEA models assume that DMUs consume the inputs and produce the outputs in the same reporting period. However, the research activities produce the outputs with a time lag. In this study, we consider the problem of measuring performance with DEA where there may be a time lag between the production of outputs and the consumption of inputs. Dual-MPO model was suggested to capture this time lag and to measure efficiency of research activity. The proposed model can be used to obtain target values of input attributes for any inefficient DMU to become efficient DMU in the case with time lag effect.

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