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# Stress Intensity Factors for Plates with Collinear and Non-Aligned Straight Cracks

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***Abstract*—**Multi-site damage (MSD) has been a challenge to aircraft, civil and power plant structures. In real life components are subjected to cracking at many vulnerable locations such as the bolt holes. However, we do not consider for the presence of multiple cracks. Unlike components with a single crack, these components are difficult to predict. When two cracks approach one another, their stress fields influence each other and produce enhancing or shielding effect depending on the position of the cracks. In the present study, numerical studies on fracture analysis have been conducted by using the developed code based on the modified virtual crack closure integral (MVCCI) technique and finite element analysis (FEA) software ABAQUS for computing SIF of plates with multiple cracks. Various parametric studies have been carried out and the results have been compared with literature where ever available and also with the solution, obtained by using ABAQUS. By conducting extensive numerical studies expressions for SIF have been obtained for collinear cracks and non-aligned cracks.

***Keywords*—**Crack interaction, Fracture mechanics, Multiple site damage, stress intensity factor, collinear cracks, non-aligned cracks.

## I. Introduction

IN real life, all components/structures are subjected to cracking at multiple locations. Single crack is merely an idealization. The MSD phenomenon was taken as a serious issue in April 1988, when a crown section of the fuselage cracked at the rivet line in a Boeing 737 airplane of the Aloha Airlines. After investigations National Transportation Safety Board and Federal Aviation Administration of USA revealed that the presence of small cracks at multiple rivet locations in an un-bonded lap joint caused the catastrophic event. This phenomenon, referred to as widespread fatigue damage (WFD), raised concerns about the structural integrity of aging aircraft due to their long-term, high-frequency services. Later this became a hot topic of research in USA.

Usually components develop cracks at the joints (rivet holes, welds). But the usual way of handling these problems is to locate the largest crack and by idealizing the case to be a single crack situation with self-similar crack growth, which generally leads to erroneous results. But in reality all components are always subjected to cracking at several places.The size and location of one crack influences the other cracks.

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WFD is a situation in which the size and density of cracks in the structure/component are such that the structure will no longer meet its damage tolerance requirement and could catastrophically fail. The WFD has two sources namely: multiple-site damage (MSD), characterized by the simultaneous presence of fatigue cracks in the same structural element and multiple-element damage (MED), characterized by the simultaneous presence of fatigue cracks in similar adjacent structural elements.

Basically there are two approaches in fracture mechanics. They are LEFM and EPFM. The parameters studied in both these approaches differ.

The important fracture parameters studied are:

1. Stress intensity factor (SIF)
2. Strain energy release rate (SERR)
3. J-integral
4. Crack tip opening angle and crack tip opening displacement (CTOA/CTOD)
5. Plastic zone length (PZL)

The parameters studied under LEFM approach are relatively simple as they do not account for the nonlinear deformation and plasticity in the vicinity of the crack tips. LEFM relates SIF to remaining life. Among these SIF is the most extensively studied parameter.

## II. Methods for Computation of Stress Intensity Factor

A number of techniques have been suggested over the last decade for computing the SIFs where analytical solutions are not available. But many of them fail to address the crack tip singularity and hence fail to represent the real behavior of the structure. There are basically two different groups of methods for estimation of these SIFs in places where analytical solutions are not available. They are the field (displacement and stress) extrapolation techniques (local approach) and those based on energy (global approach). A coarse mesh is sufficient for the latter genre of methods. They include the *J*-integral technique, the elemental crack extension, the stiffness derivative method by Parks [1] and the energy domain integral formulation given by Lorenzi [2] and Moran et al [3]. These require special post processing techniques and it is quite difficult to separate the SIF components in case of a mixed mode fracture problem. The former genre of methods, require accurate field representations around the crack tips which require a very fine mesh and special crack tip elements. Some of those methods are presented below.

### *A. Displacement ExtrapolationTtechnique*

The displacement extrapolation method is based on the displacements evaluated around the crack tip which are a primary output of FEA program. Hence in order to obtain accurate solutions we need accurate representation of the $1/\sqrt{r}$ singularity in the displacement field. This can be obtained by the usage of special crack tip elements given by [4]-[6]. The displacements obtained around the crack tip are directly used in the analytical expressions for displacement fields around the crack tips to get back the SIFs. The quarter point elements used for this method are shown in Fig. 1.

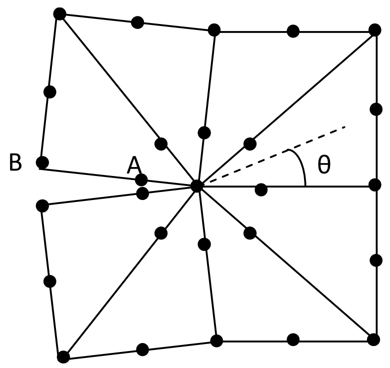


Fig. 1 Crack tip element used for displacement extrapolation method

The displacements are extracted at the nodes where they are most accurate. Kanninen et al [7] has given expression for asymptotic displacements normal to the crack plane as given in (1).

$$v = K_I\left(\frac{1+\nu}{4E}\right)\sqrt{\frac{2r}{\pi}}\left\{(2k+1)\sin\frac{\theta}{2} - \sin\frac{3\theta}{2}\right\} + \frac{A_1(1+\nu)r}{E}(k-3)\sin\theta + \frac{A_2(1+\nu)r^{\frac{3}{2}}}{E} \qquad (1)$$

Chan et al [8] sugges that the results are more accurate when extrapolation is done along the crack ($\theta = \pm\pi$). In such a case the displacements are given by (2)

$$v_A = K_I\left(\frac{(1+\nu)(k+1)}{4E}\right)\sqrt{\frac{2l}{\pi}} - \frac{A_2(1+\nu)(k+1)l^{3/2}}{12\,E} + O(l^{\frac{5}{2}}) \qquad (2)$$

$$v_B = K_I\left(\frac{(1+\nu)(k+1)}{2E}\right)\sqrt{\frac{2l}{\pi}} - \frac{A_2(1+\nu)(k+1)l^{3/2}}{12\,E} + O(l^{\frac{5}{2}}) \qquad (3)$$

From (2) and (3) the expression for SIF can be obtained as given by (4)

$$K_I = \frac{E}{3(1+\nu)(k+1)}\sqrt{\frac{2\pi}{l}}(8v_A - v_B) \qquad (4)$$

Gustavo et al [9] showed the influence of element size, element shape and mesh configuration on numerical values of $K_I$ obtained by displacement extrapolation technique and presented some guidelines to obtain $K_I$ values as good as most accurate energy based estimations within a few percent difference from the exact value

### *B. Virtual crack closure technique (VCCT)*

The method was developed by Jerram [10]. He arrived at SIF by using the crack closure integral proposed by Irwin. In this method the crack is physically extended, or closed, during two complete FEA. In the first step, the structure is analysed for the given load. Then a unit force is placed very near to the crack tip and the force required to close the crack tip to its previous configuration is evaluated. Then the work done in closing the crack is computed. The basic concept of the method is that the energy, ΔE required to extend the crack from a → a+Δa is the same as the energy required to close the crack between the same locations. This energy, which is required to open/propagate the crack is the strain energy release according to Griffith. The disadvantage in this method is that, mode separation is not possible. If the mode II energy release rate was to be found then one more step of analysis should be conducted by causing mode II crack propagation. The basic concept of the method is that the energy ΔE required to extend the crack from a → a+Δa is the same as the energy required to close the crack between the same locations. In the first step the forces at the current crack tip are evaluated. In the second step crack is assumed to have progressed by Δa and the nodal displacements are evaluated at the same location where the forces are computed previously. Then the energy ΔE is given by (5)

$$\Delta E = \frac{1}{2}(X_{1L}\Delta U_{2L} + Z_{1L}\Delta W_{2L}) \qquad (5)$$

$X_{1L}$, $Z_{1L}$ are the shear and opening forces in X and Z directions respectively.

$\Delta U_{2L}$ $\Delta W_{2L}$ are the differences in corresponding directions.

Subscript 1, 2 denote step number in which the quantity was arrived and L denotes the node to which the quantity is associated with.

### *C. Modified virtual crack closure technique (MVCCI)*

Rybicki [11] proposed the virtual crack closure technique (VCCT), which was an improved form of VCCT, called the MVCCI (modified virtual crack closure integral technique) in which only one analysis is needed. It is based on the same assumption like the VCCT. Additionally it is assumed that a crack extension of Δa will not change the forces at the crack tip too much. Hence the displacements at the node adjacent to the crack tip (free surface behind crack tip) and the forces are calculated at crack tip in the same step. Fig. 2 shows the forces and displacements to be used for evaluation of SERR.

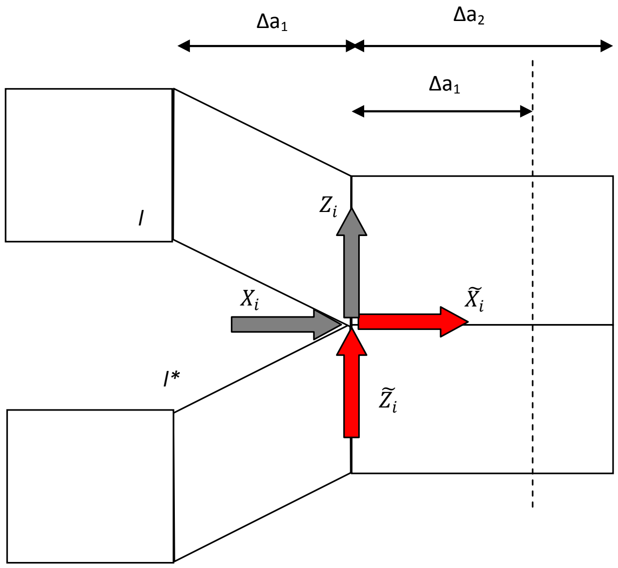


Fig. 2 Modified virtual crack closure technique for 4-noded element

The change in energy in the system is given by (6)

$$\Delta E = \frac{1}{2}(X_i' \Delta U_L + Z_i' \Delta W_L) \qquad (6)$$

The formulation explained above holds good only if Δa for the element in front of the crack tip and behind are of equal length. If automatic mesh generators are used to create complex models, the lengths will not be equal and hence, this assumption will no longer be valid. Hence some corrections are required. Rybicki [11] gave these corrections by assuming a 1/√r singularity of the stress field at the crack tip. A sketch of a crack tip modeled with two-dimensional finite elements of unequal length is shown in Fig. 2.

The forces $X_i$, $Z_i$ obtained from the finite element analysis at the crack tip (nodal point *i*) correspond to an element of length $a_2$. But what we require for the virtual crack closure technique, are the forces $\breve{X}_i$, $\breve{Z}_i$ corresponding to the relative displacements at node *l* and *l** behind the crack tip, which have been calculated for an element of length $a_1$.

The stress tip field at the crack tip can be expressed as (7)

$$\sigma(r) = \sigma_\infty . \frac{1}{\sqrt{r}} = \frac{dX}{dA} = \frac{dX}{b.dr} \qquad (7)$$

where $b$ = element thickness

$\sigma_\infty$ = undisturbed far field stress

$\sigma(r)$ = stress in front of the crack.

dX = force over a small length dr

By integrating this small force dX we get the total force X as given in (8)

$$\breve{X}_i = \int_0^{\Delta a_1} \frac{b\, \sigma_\infty dr}{r^{1/2}} = 2.b.\sigma_\infty . \Delta a_1^{1/2} \qquad (8)$$

Similarly the actual forces found can be represented by same Eq 8, where $\Delta a_1 \rightarrow \Delta a_2$ as given by (9)

$$X_i = 2.b.\sigma_\infty \Delta a_2^{1/2} \qquad (9)$$

By comparing the forces we get the following relation given by (10)

$$\breve{X}_i = \left(\frac{\Delta a_1}{\Delta a_2}\right)^{1/2} . X_i \qquad (10)$$

The forces along Z can also be corrected the same way. The corrected equations for the energy release rate components are given by (11), (12)

$$G_I = -\frac{1}{2\Delta a_1}.Z_i.(w_l - w_{l^*}).\left(\frac{\Delta a_1}{\Delta a_2}\right)^{1/2} \qquad (11)$$

$$G_{II} = -\frac{1}{2\Delta a_1}.X_i.(u_l - u_{l^*}).\left(\frac{\Delta a_1}{\Delta a_2}\right)^{1/2} \qquad (12)$$

The correction can also be made to the displacements which were computed for a length $\Delta a_1$, to match the forces $X_i$, $Z_i$ at the crack tip, which were computed for an element length $\Delta a_2$. This method is pretty simple but has a limitation as the displacement variation can be only in the order of the polynomial being used for the shape functions. For a linear quadrilateral element it can be approximated by simple linear interpolation. The corrected equations for the energy release rate components are given by (13), (14):

$$G_I = -\frac{1}{2\Delta a_2}.Z_i.(w_l - w_{l^*}).\frac{\Delta a_2}{\Delta a_1} \qquad (13)$$

$$G_{II} = -\frac{1}{2\Delta a_2}.X_i.(u_l - u_{l^*}).\frac{\Delta a_2}{\Delta a_1} \qquad (14)$$

The method first described imposes an analytical relationship based on the $1/\sqrt{r}$ singularity of the stress field at the crack tip. However, the second method does not take into account of the square root singularity.

## III. Numerical Studies

Several plate panels have been modeled with different crack configurations. The horizontal and vertical spacing between the lead crack and the secondary crack have been varied and solved by both the MVCCI technique and ABAQUS. The results are then compared. Three example problems, namely

(i) Plate panel with centre crack

(ii) Plate panel with collinear multiple cracks with different spacing

(iii) Plate panel with non-aligned multiple cracks with different horizontal and vertical spacing

All plate panels are subjected to a uniaxial stress of 77.78 MPa which corresponds to 35 kN load. The problems have been solved by the FEA code developed in MATLAB based on the MVCCI technique and also by using the FEA software ABAQUS. The same mesh that has been used in ABAQUS was imported and used in the developed code to ensure that that the meshing does not influence the difference in solution.

### *A. Finite element modeling*

Finite element modeling of the panels has been carried out by employing the crack tip elements and regular quadrilateral elements. The crack was represented by using two set of nodes of identical coordinates at the upper face and lower face of the crack respectively. The SIF obtained by using both types of crack tip elements have been compared with the SIF obtained by using regular four noded linear quadrilateral elements. The results are shown in Table. I. It is observed that the usage of crack tip elements did not make any significant difference.

Hence, normal elements have been used for the analysis that follows. But a finer mesh was used around the crack tip.

TABLE I
SIF OBTAINED BY USING VARIOUS TYPES OF ELEMENTS

| Crack width (mm) | Stress intensity factor (MPa √mm) obtained by | | |
|---|---|---|---|
| | Quarter point element | Collapsed quadrilateral element | Regular 4-noded linear element |
| 15 | 246.5 | 246.5 | 246.2 |
| 20 | 285.9 | 286.0 | 283.9 |

To further show the efficiency of the present model in computation of SIF, some standard benchmark problems have been solved and compared with the existing literature. It is also compared with the solutions obtained from commercial software ABAQUS. Static analysis has been performed to compute SIF for different configurations to compare the solutions obtained with the developed code in order to validate it. The same mesh pattern has been used in both ABAQUS and MATLAB code. They are presented below in the following sections.

*B. Plate with a centre crack*

The problem of a plate with a centre crack is solved. Special crack tip elements have not been as the method is based on the global strain energy change rather than a localized parameter. A total of 6191 elements have been used for the centre crack problem. The details of the problem are given below.

σ = 50 MPa
D = 250 mm
2W = 150 mm
2a = 12 mm

SIF value obtained by using analytical expression is 219.070 $MPa\sqrt{mm}$

SIF value obtained by using MVCCI method is 217.715 $MPa\sqrt{mm}$

The present method is found to be in good agreement with the existing solution. So, further numerical simulations have been performed on plate specimen with multiple crack configurations.

*C. Plate with two collinear cracks*

A plate with two cracks of the same length which are parallel and collinear have been modeled as shown in Fig. 3, in commercial code ABAQUS and the MATLAB code by employing the MVCCI method. The cracks have been placed at different horizontal spacing and solved.

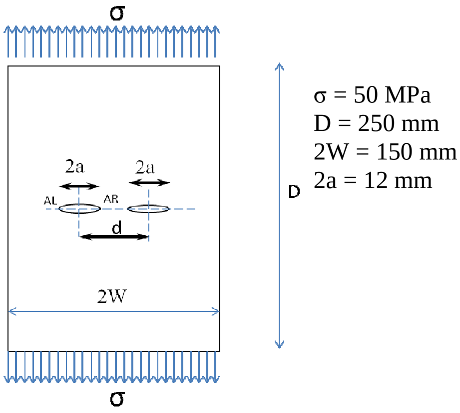


Fig. 3 Plate with two collinear cracks

d= centre to centre spacing between cracks in horizontal direction

For all the simulations the elements used are the regular 4-noded quadrilateral elements. Each node contains two degrees of freedom, namely the translations $U_x$ and $U_y$. The same elements have been used both in ABAQUS and the MATLAB code. The same type of mesh has been used. Very fine mesh has been used around the crack tips. Meshing was similar to the case of a plate with centre crack. About 3410 elements (for 2a/d = 0.2) to 7042 elements (for 2a/d = 0.6) have been used. More number of elements has been used as the crack spacing is reduced. The normalized SIF ($k_I/\sigma\sqrt{\pi a}$) values obtained by using the present model are compared with the values obtained by using ABAQUS and the solution obtained by Erdogan [12] are presented in Table. II.

The results show that the present method is in close agreement with those of the available literature and ABAQUS software. The variation of SIF is shown in Fig. 4 and Fig. 5

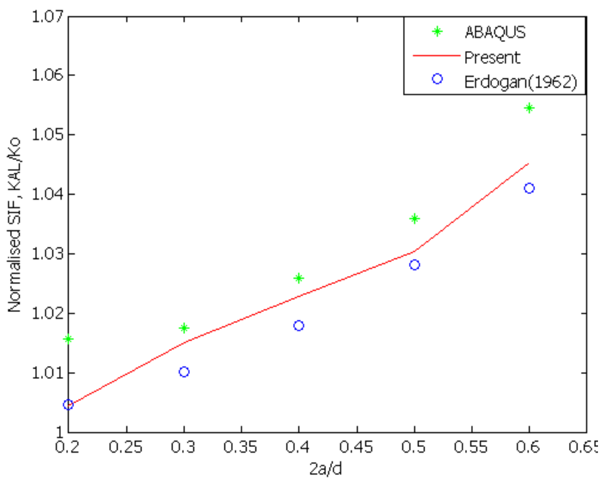


Fig. 4 Normalized SIF (KAL/Ko) vs 2a/d for crack tip AL

TABLE II
COMPARISON OF NORMALIZED SIF FOR PLATE WITH TWO COLLINEAR CRACKS

| 2a/d | $k_I/\sigma\sqrt{\pi a}$ at Tip AL | | | $k_I/\sigma\sqrt{\pi a}$ at Tip AR | | |
|---|---|---|---|---|---|---|
| | **Abaqus** | **Present method** | **Erdogan [12]** | **Abaqus** | **Present method** | **Erdogan [12]** |
| **0.2** | 1.01575 | 1.00450 | 1.00462 | 1.01852 | 1.00817 | 1.00566 |
| **0.3** | 1.01760 | 1.01510 | 1.01016 | 1.02266 | 1.01792 | 1.01383 |
| **0.4** | 1.02589 | 1.02278 | 1.01787 | 1.04017 | 1.03217 | 1.02717 |
| **0.5** | 1.03602 | 1.03031 | 1.02795 | 1.05583 | 1.05214 | 1.04796 |
| **0.6** | 1.05445 | 1.04533 | 1.04094 | 1.09222 | 1.08261 | 1.08040 |

*AL refers to the left tip of crack A and AR, the right tip of crack A. They are as indicated in Fig. 3

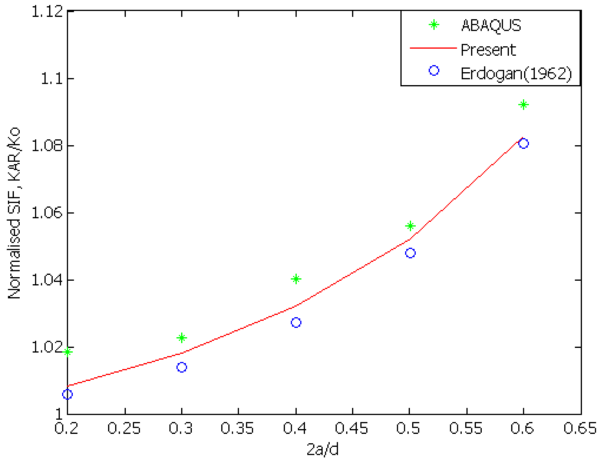


Fig. 5 Normalized SIF (KAR/Ko) vs 2a/d for crack tip AR

The Values of the present study are found to be in good agreement.

*D. Plate with two non-aligned cracks*

The numerical and experimental studies on multiple cracks are available in all the literature deal with two or more collinear cracks. Hence, non-aligned cracks of different lengths are also considered in the present investigations.

Using the MATLAB code FEA has been performed to obtain the forces and displacements and then the nodal information has been post processed to obtain SIF at the various tips. A plate of 150mm x 250 mm was considered for analysis. Two cracks of different lengths 20mm and 15mm have been considered. The details of the specimen considered are shown in Fig. 6

Fine mesh of about 1mm has been used around the crack region. Then it has been gradually increased to about 20mm on the top and bottom edges. SIF found is normalized w. r. t SIF solution of plate with a single crack of the corresponding crack's dimension in order to study the relative percentage increase in SIF. The spacing between the cracks has been varied from 0 to 25mm both in horizontal and vertical directions.

The variation of SIF along vertical spacing (Y), for all the four crack tips AL (left tip of crack A), AR (right tip of crack A), BL (left tip of crack B) and BR (right tip of crack B) for mode I and mode II are plotted for different horizontal spacing (H) in Figs. 7 to Fig. 22.

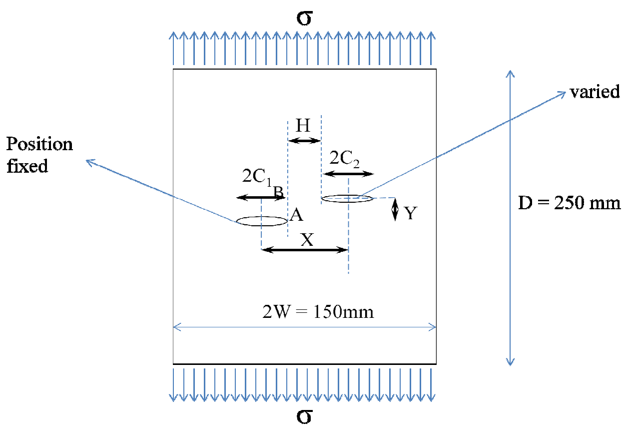


Fig. 6 Plate with non-aligned cracks

$2C_1$ = 20mm
$2C_2$ = 15mm
2W = 150mm
$\sigma$ = 77.78 MPa (Corresponding to 35 kN and 0.459 $f_y$)

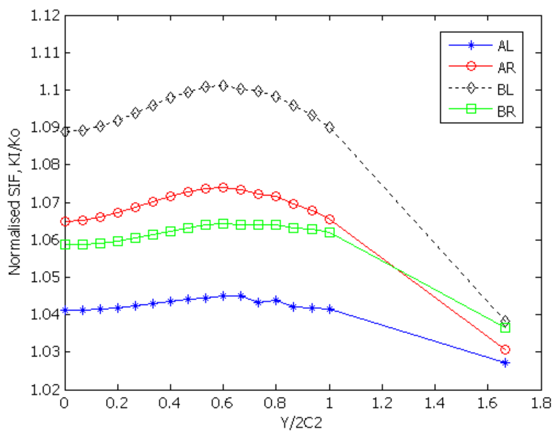


Fig. 7 Normalized SIF ($K_I/K_o$) vs $Y/2C_2$ for H=15

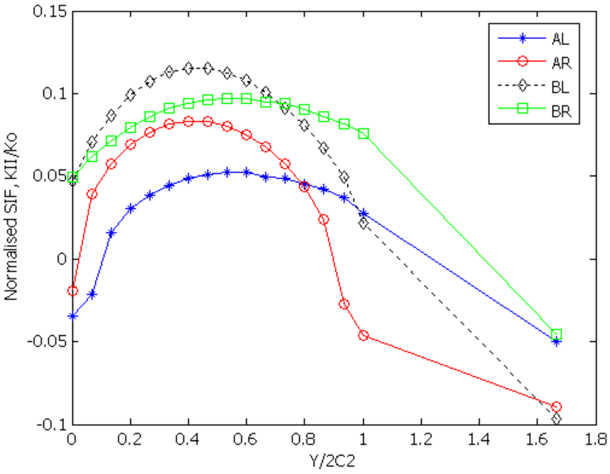


Fig. 8 Normalized SIF ($K_{II}/K_o$) vs $Y/2C_2$ for H=15

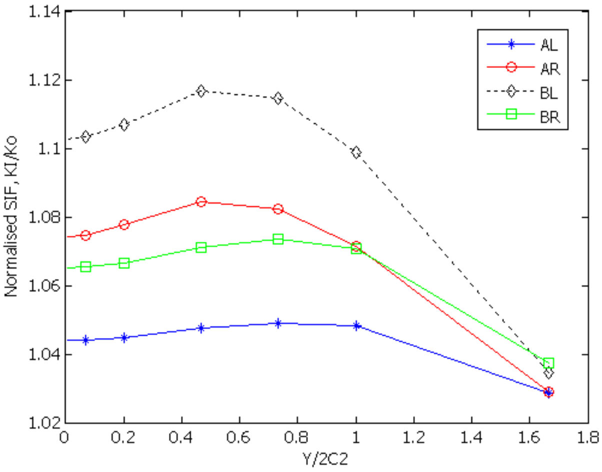


Fig. 9 Normalized SIF ($K_I/K_o$) vs $Y/2C_2$ for H=13

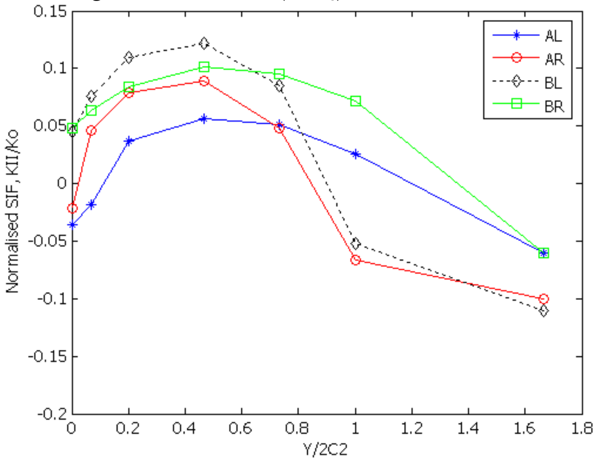


Fig. 10 Normalized SIF ($K_{II}/K_o$) vs $Y/2C_2$ for H=13

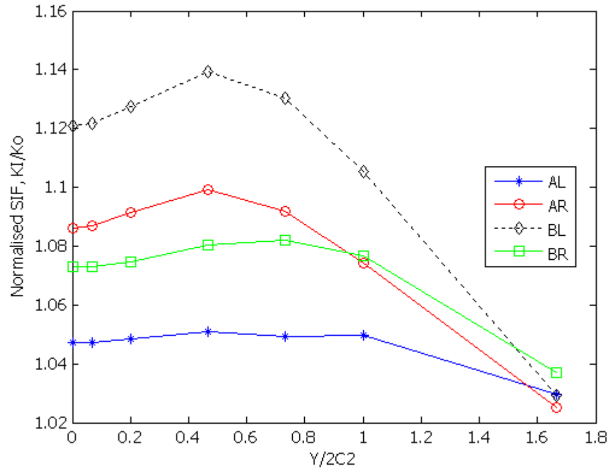


Fig. 11 Normalized SIF ($K_I/K_o$) vs $Y/2C_2$ for H=11

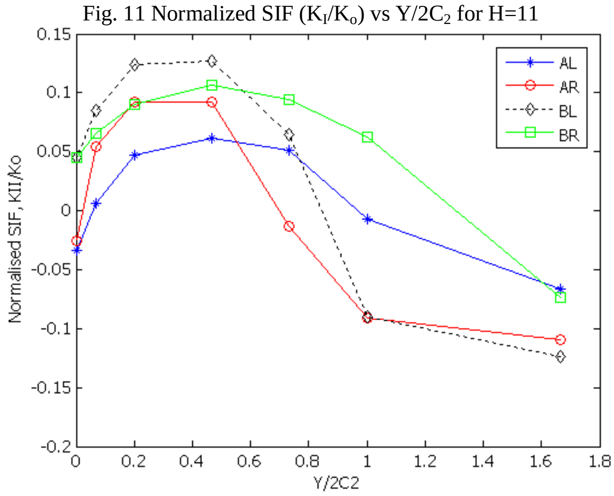


Fig. 12 Normalized SIF ($K_{II}/K_o$) vs $Y/2C_2$ for H=11

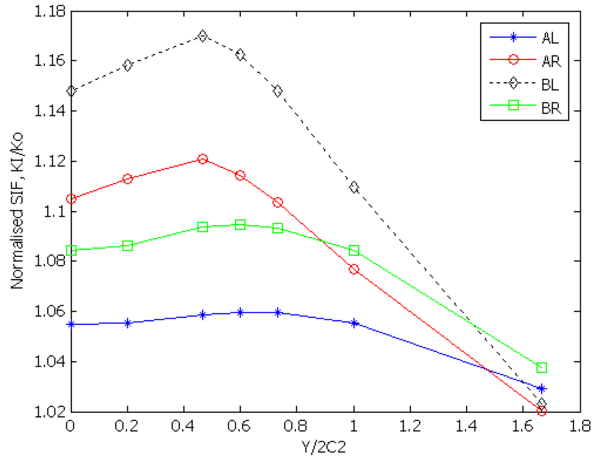


Fig. 13 Normalized SIF ($K_I/K_o$) vs $Y/2C_2$ for H=9

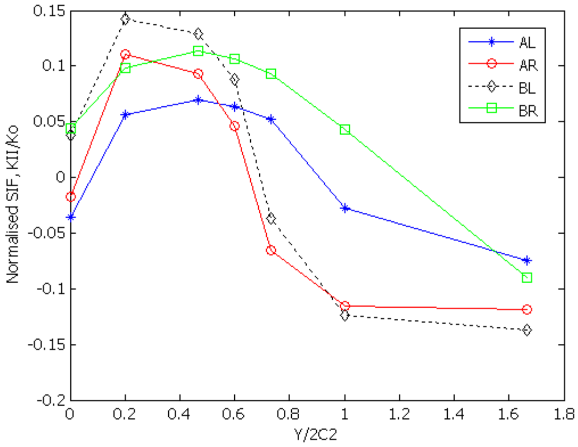


Fig. 14 Normalized SIF ($K_{II}/K_o$) vs $Y/2C_2$ for H=9

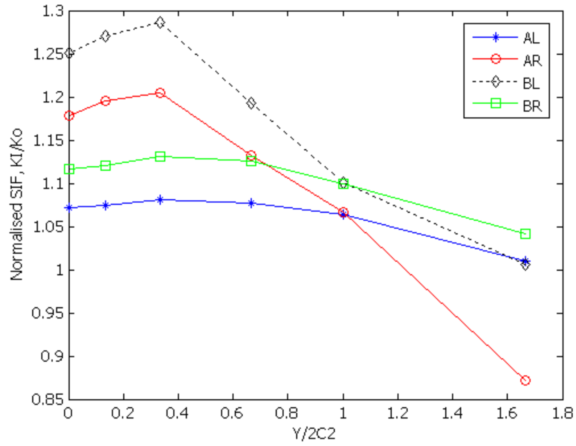


Fig. 17 Normalized SIF ($K_I/K_o$) vs $Y/2C_2$ for H=5

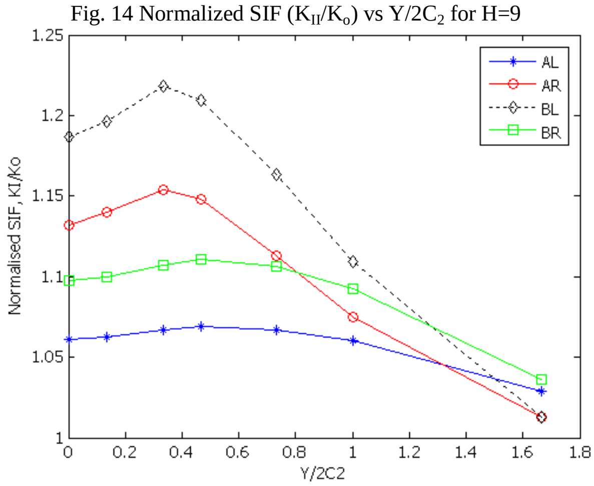


Fig. 15 Normalized SIF ($K_I/K_o$) vs $Y/2C_2$ for H=7

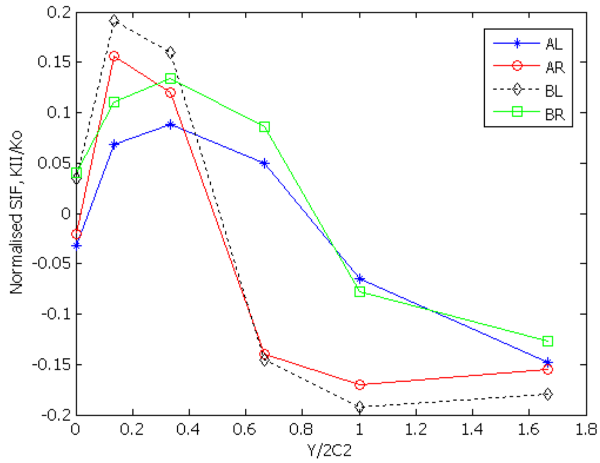


Fig. 18 Normalized SIF ($K_{II}/K_o$) vs $Y/2C_2$ for H=5

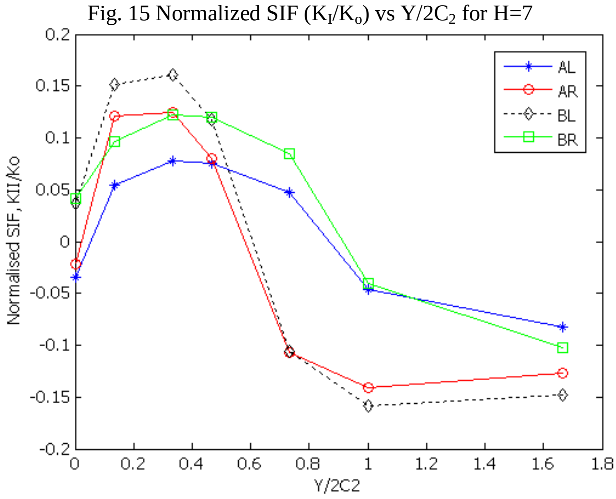


Fig. 16 Normalized SIF ($K_{II}/K_o$) vs $Y/2C_2$ for H=7

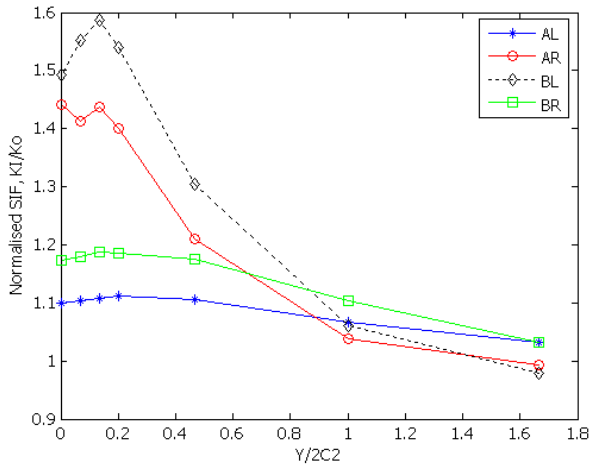


Fig. 19 Normalized SIF ($K_I/K_o$) vs $Y/2C_2$ for H=2

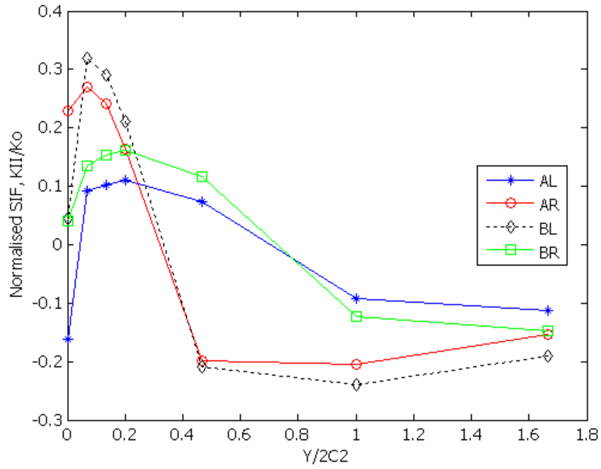


Fig. 20 Normalized SIF ($K_{II}/K_o$) vs $Y/2C_2$ for H=2

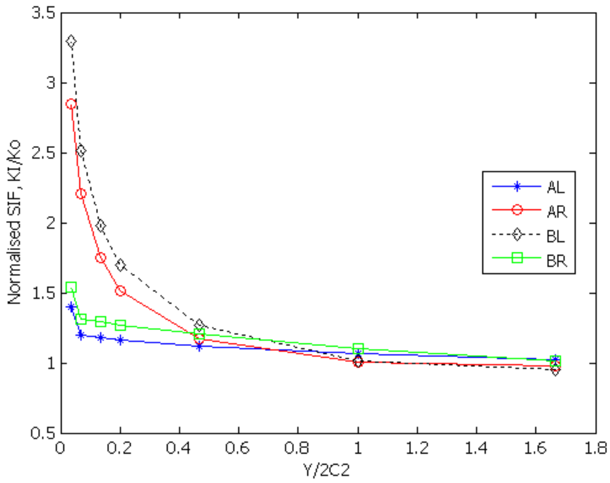


Fig. 21 Normalized SIF ($K_I/K_o$) vs $Y/2C_2$ for H=0

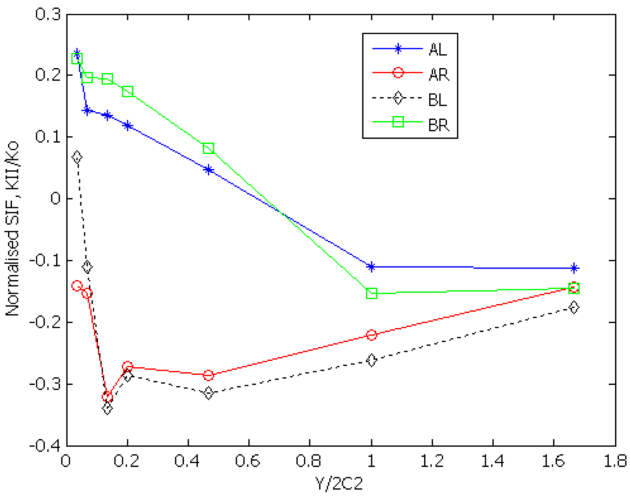


Fig. 22 Normalized SIF ($K_{II}/K_o$) vs $Y/2C_2$ for H=0

## IV. Discussion of Results

The SIF has been computed for different cases of multiple cracks. The SIF values are found to be in close agreement with the results from ABAQUS. It is found that the SIF does not increase continuously with decrease in the spacing between the tips. With constant horizontal spacing by varying the vertical spacing it has been found that the SIF increases to a maximum at a particular distance and then reduces. This distance at which the peak SIF is obtained is also not a constant. It changes with the vertical spacing between the cracks. The horizontal spacing is found to have a dominant effect over the vertical spacing. SIF increases as much as 60% in case of non-aligned cracks and upto around 330% for collinear cracks.

Several other cases of collinear cracks have also been analyzed by varying the size of the secondary cracks ($2C_2$ = 5mm, 10mm, 15mm). The horizontal spacing has been varied from H = 2mm to 40 mm (H/W =0.0266 to 0.533). Beyond this the variation of H is not found to influence the value of SIF. If the spacing between the cracks are further reduced then it might cause the cracks to link up (the plastic zones will come into contact). So, the values of SIF will not be valid. Hence this range has been chosen for present study. SIF was found to vary in exponential way when the spacing between cracks is reduced. Based on the results obtained a curve fit has been done and expressions have been arrived at for collinear case of cracks for computation of SIF. The expressions for interaction factors (IF) have been presented.

1. $K_{1,AR} = K_{0,AL} \times IF$

$K_{1,AR}$ = SIF at tip AR due to MSD

$K_{0,AR}$ = SIF at tip AR in a plate with centre crack of same dimensions as given by (15), (16)

$$IF = ae^{\frac{\{f(y)\}}{\{f(x)\}}} \qquad (15)$$

$$f(y) = by + cy^2 + dy^3 + ey^4 + fy^5$$

$$f(x) = gx^{0.5} + hx + ix^2 + jx^3$$

where,

$y = 2C_2 / W$

$x = H / W$

| | |
|---|---|
| a | 1.006 |
| h | 3.396 |
| i | 0.5874 |
| j | -0.8902 |
| b | 0.1893 |
| c | 0.6758 |
| d | -2.138 |
| e | 4.192 |
| f | -8.611 |
| g | 0.4153 |

SSE: 0.0003039

R-square: 0.9983

Adjusted R-square: 0.9975

RMSE: 0.004501

2. $K_{1,AL} = K_{0,AL}$ x IF

$K_{1,AL}$ = SIF at tip AL due to MSD

$K_{0,AL}$ = SIF at tip AL in a plate with centre crack of same dimensions

$$IF = ae^{\left\{\frac{f(y)}{f(x)}\right\}} \quad (16)$$

$$f(y) = b + cy + dy^2 + ey^3 + fy^4 + gy^5$$

$$f(x) = h + ix + jx^2 + kx^3$$

$y = 2C_2 / W$

$x = H / W$

| | |
|---|---|
| a | 1.0122 |
| b | .0004216 |
| c | -0.003862 |
| d | 0.6389 |
| e | -1.808 |
| f | 1.134 |
| g | 4.548 |
| h | 0.1025 |
| i | 2.254 |
| j | -2.473 |
| k | 4.059 |

SSE: 3.003e-005
R-square: 0.9974
Adjusted R-square: 0.9958
RMSE: 0.001465

Several cases of non-aligned cracks have also been studied. The horizontal spacing has been varied from 2mm to 25mm. The vertical spacing has also been varied from 0 mm to 25 mm. Based on the obtained results a surface fit has been done. The expression arrived at for computation of SIF is given by (17), (18)

1. $K_{1,AL} = K_{0,AL}$ x IF

$$IF = ae^{\left\{\frac{f_1(x,y)}{f_2(x,y)}\right\}} \quad (17)$$

$$f1(x,y) = bx + cy + dxy + ex^2 + fy^2$$

$$f_2(x,y) = 1 + gx + hy + ixy + jx^2 + ky^2$$

where, $x = H / 2C_2$, $y = V / 2C_2$

| | |
|---|---|
| a | 1.221 |
| b | -2.778 |
| c | -0.4515 |
| d | 2.291 |
| e | -6.58 |
| f | -1.307 |
| g | 21.69 |
| h | 8.656 |
| i | -14.54 |
| j | 35.3 |
| k | 3.376 |

SSE: 0.0001066
R-square: 0.9984
Adjusted R-square: 0.9981
RMSE: 0.001539

2. $K_{1,AR} = K_{0,AR}$ x IF

$$IF = ae^{\left\{\frac{f_1(x,y)}{f_2(x,y)}\right\}} \quad (18)$$

$$f_1(x) = bx + cy + dxy + ex^2 + fy^2$$

$$f_2(x) = 1 + gx + hy + ixy + jx^2 + ky^2$$

where, $x = H / 2C_2$, $y = V / 2C_2$

| | |
|---|---|
| b | -12.63 |
| c | 6.487 |
| g | 39.3 |
| h | 12.83 |
| d | 35.53 |
| i | -31.08 |
| a | 2.361 |
| e | -316.2 |
| f | -134.8 |
| j | 368.4 |
| k | 139.3 |

SSE: 0.0007883
R-square: 0.9996
Adjusted R-square: 0.9996
RMSE: 0.004185

The above expression does not account for inclined cracks. It can provide SIF for the main crack tips which should be straight. It is also correct only as long as the projection of the minor crack (secondary crack) does not overlap with the main crack i.e. H > 0.

## V. Conclusion

A model has been developed in MATLAB by implementing MVCCI technique for evaluation of the strain energy release rate and SIF. A plate with centre crack has been solved by using the developed model and the results are compared with solutions obtained by using ABAQUS and those available in the literature. Many cases of collinear cracks have been solved and compared with ABAQUS solutions and those available in the literature. It is found to be in close agreement with the available results. A number of multi crack problems have been solved and the results are shown. Based on the solutions from the extensive numerical simulations curve fitting has been done for collinear cracks and non-aligned cracks and hence expressions have been proposed for evaluation of SIF of multiple cracks.

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