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Open Problems on Zeros of Analytic Functions in Finite Quantum Systems

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Abstract—The paper contains an investigation on basic problems about the zeros of analytic theta functions. A brief introduction to analytic representation of finite quantum systems is given. The zeros of this function and there evolution time are discussed. Two open problems are introduced. The first problem discusses the cases when the zeros follow the same path. As the basis change the quantum state $|f\rangle$ transforms into different quantum state. The second problem is to define a map between two toruses where the domain and the range of this map are the analytic functions on toruses.

Keywords—Open problems, constraint, change of basis.

I. Introduction

THIS Paper is devoted to discuses some problem related to the paths of zeros in analytic representation of finite quantum systems on a torus. Analytic functions are considered from [1], [2], [3] and used in various places in physic sciences. The analytic Bargmann function [4], [5], [6], [7], [8], [9], [10] is The most famous one. Ref [12] has considered analytic representations of finite quantum systems on a torus. The analytic function has exactly $\mathfrak{N}$ zeros. Ref [13] has been studied the motion of the zeros.In some cases $\mathfrak{N}$ of the zeros follow the same path and in other cases each zero follow its own path.It is seen that the same zeros with two different Hamiltonian, in the first case follow the same path and in the second case follow different paths. Also we have seen that for the same Hamiltonian, two sets of zeros, follow the same path and different paths correspondingly. We concluded that there is specific constraint should be satisfied, and either the zeros or the Hamiltonian subjected to the constraint which should involve both the zeros and the Hamiltonian. The first problem is what is the constraint. A unitary transformation is equivalent to a change of basis. We try to discuss how to define a map from torus to another such that the domain of this map is the zeros of analytic function in first torus and the range is the zeros of analytic function in second torus. The second problem is what is the definition of the map.

II. Zeros of analytic representation of finite quantum systems

Let H be a Hilbert space with dimension $\mathfrak{N}$ and let $|X_m\rangle, |P_m\rangle$ be the position states and momentum states respectively($m \in \mathfrak{N}$) where

$$|P_m\rangle = \mathbb{F}|X_m\rangle = \mathfrak{N}^{-1/2}\sum_n (\exp\left[i\frac{2\pi m}{\mathfrak{N}}\right])|X_m\rangle, \quad (1)$$

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and $\mathbb{F}$ the Fourier operator:

$$\mathbb{F} = \mathfrak{N}^{-1/2}\sum_{m,n}(\exp\left[i\frac{2\pi m}{\mathfrak{N}}\right])|X_m\rangle\langle X_n|. \quad (2)$$

The position and momentum operators are defined as

$$x = \sum_{n=0}^{\mathfrak{N}-1} n|X_n\rangle\langle X_n|; \quad p = \mathbb{F}x\mathbb{F}^\dagger = \sum_{n=0}^{\mathfrak{N}-1} n|P_n\rangle\langle P_n|, \quad (3)$$

respectively
We study an arbitrary normalized state $|\mathbb{F}\rangle$

$$|\mathbb{F}\rangle = \sum_m \mathbb{F}_m|X_m\rangle; \quad \sum_m |\mathbb{F}_m|^2 = 1, \quad (4)$$

Following ref [12], [13] we introduce the analytic representations of finite quantum systems on a torus.
We represent the state $|\mathbb{F}\rangle$ of Eq.(4), with the analytic function

$$f(z) = \pi^{-1/4}\sum_{m=0}^{\mathfrak{N}-1}\mathbb{F}_m\vartheta_3[\pi m\mathfrak{N}^{-1} - z\sqrt{\frac{\pi}{2\mathfrak{N}}}; i\mathfrak{N}^{-1}] \quad (5)$$

which satisfy quasi-periodic condition

$$\begin{aligned} f\left[z + \sqrt{2\pi\mathfrak{N}}\right] &= f(z) \\ f\left[z + i\sqrt{2\pi\mathfrak{N}}\right] &= f(z)\exp\left[\pi\mathfrak{N} - i(2\pi\mathfrak{N})^{1/2}z\right] \end{aligned} \quad (6)$$

where ϑ_3 is Theta function and defined as

$$\vartheta_3(u,\tau) = \sum_{n=-\infty}^{\infty}\exp(i\pi\tau n^2 + i2nu). \quad (7)$$

The analytic function $f(z)$ is defined on a cell $[x_0, x_0 + \sqrt{2\pi\mathfrak{N}}) \times [x_1, x_1 + \sqrt{2\pi\mathfrak{N}})$ (on a torus)
The sum of the zeros $\mathfrak{z}_n$ of analytic function $f(z)$ is

$$\sum_{n=1}^{\mathfrak{N}}\mathfrak{z}_n = (2\pi)^{1/2}\mathfrak{N}^{3/2}(l + ir) + \left(\frac{\pi}{2}\right)^{1/2}\mathfrak{N}^{3/2}(1+i), \quad (8)$$

where l, r are integers.
Ref.[12], [13] has constructed the function $f(z)$ from its zeros $\mathfrak{z}_n$ as following:
Let $\mathfrak{z}_n$ be the zeros of the analytic function $f(z)$ and suppose that this zeros satisfy the relation.(8) then the analytic function $f(z)$ is defined as

$$\begin{aligned} f(z) &= q\exp\left[-i\left(\frac{2\pi}{\mathfrak{N}}\right)^{1/2}lz\right]\prod_{n=1}^{\mathfrak{N}}\vartheta_3\left[w_n(z);\ i\right] \\ w_n(z) &= \left(\frac{\pi}{2\mathfrak{N}}\right)^{1/2}(z - \mathfrak{z}_n) + \frac{\pi(1+i)}{2} \end{aligned} \quad (9)$$

where l is the integer relation of Eq.(8) and q is a constant calculated from the normalization condition.

We consider the state $|\mathbb{F}(0)\rangle = \sum \mathbb{F}_m(0)|X;m\rangle$ at $t = 0$. Using the Hamiltonian H, the state $|\mathbb{F}(0)\rangle$ evolves at time t

$$|\mathbb{F}(t)\rangle = \exp(itH)|\mathbb{F}(0)\rangle = \sum_{m=0}^{\mathfrak{N}-1} \mathbb{F}_m(t)|X_m\rangle \tag{10}$$

numerically we calculate the zeros $\mathfrak{z}_n$ of $f(z)$.
Ref.[13] has discussed the Periodic finite quantum systems. In some cases $\mathfrak{N}$ of the zeros follow the same path. We say that this path has multiplicity d.

III. Constraints on the zeros of the functions $f(z)$

It is will known that in the periodic systems the $\mathfrak{N}$ paths of the zeros $\mathfrak{z}_n(t)$ are in general closed curves on the torus. In some cases $\mathfrak{N}$ of the zeros follow the same path and in other cases each zero follow its own path.
We discuss how the same zeros with two different Hamiltonian, follow the same path with the first Hamiltonian and different paths with the other Hamiltonian. We consider two sets of zeros with one Hamiltonian, one of them follow same path and the other follow different paths. Therefore there is specific constraint should be satisfied, and either the zeros or the Hamiltonian subjected to the constraint.

Example 1:
Let $\mathfrak{z}_0(t), \mathfrak{z}_1(t), \mathfrak{z}_2(t)$ be the paths of the three zeros. We assume that the initial values of these zeros are

$$\mathfrak{z}_0(0) = 2 + 2i,\ \mathfrak{z}_1(0) = 2 + 2.5i,\ \mathfrak{z}_2(0) = 2.5 + 2i. \tag{11}$$

We consider two different Hamiltonians.
◇ The first Hamiltonian is

$$H = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{12}$$

This Hamiltonian has the eigenvalues $0, 1, 2$ with period $\mathcal{O} = 2\pi$.
◇ The second Hamiltonian is

$$H = \begin{bmatrix} 2 & -i & 0 \\ i & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{13}$$

This Hamiltonian has the eigenvalues $0, 1, 2$ with period $\mathcal{O} = 2\pi$.
In the case of Hamiltonian.(12) we found numerically that

$$\mathfrak{z}_0(\mathcal{O} + t) = \mathfrak{z}_1(t),\ \mathfrak{z}_1(\mathcal{O} + t) = \mathfrak{z}_2(t),\ \mathfrak{z}_2(\mathcal{O} + t) = \mathfrak{z}_0(t). \tag{14}$$

After period the three zeros follow the same path. In Fig.1 we present the path of these zeros.
In the case of Hamiltonian.(13), after period each zero follows its own path. In Fig.2 we present the path of these zeros.
It is seen that the same zeros with two different Hamiltonian, in the first case follow the same path and in the second case follow different paths.

Example 2:
We consider the Hamiltonian

$$H = \begin{bmatrix} 1 & -i & 0 \\ i & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \tag{15}$$

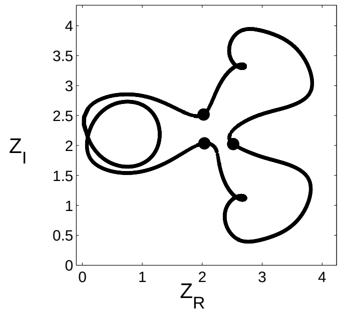

Fig. 1. The path of the zeros $\mathfrak{z}_0(t)$, $\mathfrak{z}_1(t)$, $\mathfrak{z}_2(t)$ for the system with the Hamiltonian of Eq.(12). All zeros follow the same path. At $t = 0$ the zeros $\mathfrak{z}_0(0)$, $\mathfrak{z}_1(0)$, $\mathfrak{z}_2(0)$ are given in Eq.(11)

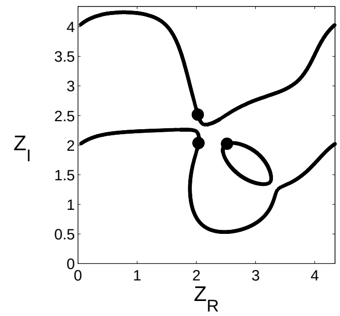

Fig. 2. The path of the zeros $\mathfrak{z}_0(t)$, $\mathfrak{z}_1(t)$, $\mathfrak{z}_2(t)$ for the system with the Hamiltonian of Eq.(13). Each zero follows its own path. At $t = 0$ the zeros $\mathfrak{z}_0(0)$, $\mathfrak{z}_1(0)$, $\mathfrak{z}_2(0)$ are given in Eq.(11) .

The Hamiltonian has the eigenvalues $0, 2, 2$. Here we consider two cases.
◇ In the first case, the initial values of the zeros are

$$\mathfrak{z}_0(0) = 1.5 + 1.5i,\ \mathfrak{z}_1(0) = 2 + 3i,\ \mathfrak{z}_2(0) = 3 + 2i, \tag{16}$$

with period $\mathcal{O} = 2\pi$.
◇ In the second case, the initial values of the zeros are

$$\mathfrak{z}_0(0) = 2.1 + 2.1i,\ \mathfrak{z}_1(0) = 1.4 + 3.4i,\ \mathfrak{z}_2(0) = 3 + 0.1i, \tag{17}$$

with period $\mathcal{O} = 2\pi$.
In the case of Eq.(16)one can see that

$$\mathfrak{z}_0(\mathcal{O} + t) = \mathfrak{z}_2(t), \quad \mathfrak{z}_2(\mathcal{O} + t) = \mathfrak{z}_0(t) \tag{18}$$

Here the zeros $\mathfrak{z}_0$, $\mathfrak{z}_2$ follow the same path and the third zero follows its own path. This is shown in Fig.3 .
In the case of Eq.(17) each zero follows its own path. This is shown in Fig.4.
For the Hamiltonian (15), we get two of the zeros in the case of Eq.(16) follow the same path and in the case of Eq.(17) each zero follows different path. It is seen that for the same Hamiltonian, two sets of zeros, follow the same path and different paths correspondingly.We concluded that there is specific constraint should be satisfied, and either the zeros or the Hamiltonian subjected to the constraint. Therefore if there is such constraint, it should involve both the zeros and the Hamiltonian.

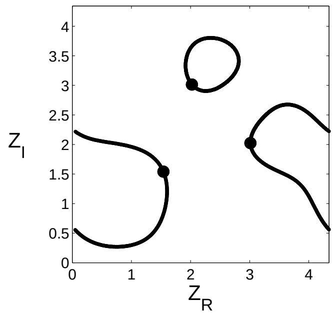

Fig. 3. The path of the zeros $\mathfrak{z}_0(t)$, $\mathfrak{z}_1(t)$, $\mathfrak{z}_2(t)$ for the system with the Hamiltonian of Eq.(15). the zeros $\mathfrak{z}_0, \mathfrak{z}_2$ follow the same path and the third zero follows its own path. At $t = 0$ the zeros $\mathfrak{z}_0(0)$, $\mathfrak{z}_1(0)$, $\mathfrak{z}_2(0)$ are given in Eq.(16)

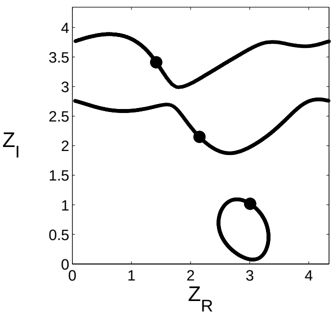

Fig. 4. The path of the zeros $\mathfrak{z}_0(t)$, $\mathfrak{z}_1(t)$, $\mathfrak{z}_2(t)$ for the system with the Hamiltonian of Eq.(15). Each zero follows its own path. At $t = 0$ the zeros $\mathfrak{z}_0(0)$, $\mathfrak{z}_1(0)$, $\mathfrak{z}_2(0)$ are given in Eq.(17) .

Problem 1

We consider the Hamiltonian H with the initial zeros $\mathfrak{z}_1, \mathfrak{z}_2, ..., \mathfrak{z}_{\mathfrak{N}}$.
Is there any constraint such that D of these zeros follow same path ?
If there is such constraint, what is that constraint?

IV. The Change of the Basis

The unitary transformation is one-to-one function between two Helbert spaces. Let A be a Hermitean matrix, and let U be a unitary transformation. It is will known that the matrix $UAU^\dagger$ is Hermation and has the same of eigenvalues of A. A unitary transformation is equivalent to a change of basis. It is a transformation that transforms one basis into another. As an example of a unitary transformation we consider the Symplectic transformations $\mathfrak{U}$.

Symplectic transformations

Following ref.[11] we introduce the Symplectic transformations $\mathfrak{U}$ in the $Z_{\mathfrak{N}} \times Z_{\mathfrak{N}}$ phase-space of a finite quantum system. We consider the unitary transformations

$$\begin{aligned} X^{'} &= \mathfrak{U} X \mathfrak{U}^\dagger = X^\kappa Z^\lambda \psi(2^{-1}\kappa\lambda), \\ Z^{'} &= \mathfrak{U} Z \mathfrak{U}^\dagger = X^\mu Z^\nu \psi(2^{-1}\mu\nu), \end{aligned} \quad (19)$$

Here

$$X = \exp[\frac{-i2\pi p}{\mathfrak{N}}],\ Z = \exp[\frac{i2\pi x}{\mathfrak{N}}],\ \psi(a) = \exp\left[i\frac{2\pi a}{\mathfrak{N}}\right] \quad (20)$$

where x, p are the position and momentum operators and $\lambda, \kappa, \mu, \nu$ are integers in $Z_{\mathfrak{N}}$ obey the relation

$$\kappa\nu - \lambda\mu = 1(mod(\mathfrak{N})). \quad (21)$$

By reference to ref.[11] we construct the unitary operator $\mathfrak{U}$.

Example 3:
We consider a three-dimensional Hilbert space ($\mathfrak{N} = 3$) and $\mathfrak{U}(1, -1\ -\ 1)$,which leads (by definition in Eq.(19)) to the transformations

$$\begin{aligned} X^{'} &= \mathfrak{U} X \mathfrak{U}^\dagger = X Z^{-1} \omega(-\frac{1}{2}), \\ Z^{'} &= \mathfrak{U} Z \mathfrak{U}^\dagger = X^{-1} Z^2 \omega(-1). \end{aligned} \quad (22)$$

The operator $\mathfrak{U}$ is given in a matrix $\mathfrak{U}(\imath, \jmath)$ and the matrix elements $\mathfrak{U}(\imath, \jmath)$ are given in table I. The transformation with

TABLE I
THE COEFFICIENTS $\mathfrak{U}(\imath, \jmath)$ FOR THE TRANSFORMATIONS OF EQ.(22).

	$\imath = 0$	$\imath = 1$	$\imath = 2$
$\jmath = 0$	0.5774	0.2887 + 0.5i	0.5774
$\jmath = 1$	-0.2887 + 0.5i	0.2887 - 0.5i	0.5774
$\jmath = 2$	0.5774	0.2887 - 0.5i	-0.2887 + 0.5i

operatore $\mathfrak{U}$ on the analytic function $f(z)$

$$f(z) = \pi^{-1/4} \sum_{m=0}^{\mathfrak{N}-1} \mathbb{F}_m\ \vartheta_3[\frac{\pi m}{\mathfrak{N}} - z\sqrt{\frac{\pi}{2\mathfrak{N}}}; \frac{i}{\mathfrak{N}}] \quad (23)$$

can be expressed as

$$\mathfrak{U} f(z) \longrightarrow \pi^{-1/4} \sum_{m=0}^{\mathfrak{N}-1} \mathfrak{U}_{ml} \mathbb{F}_l\ \vartheta_3[\frac{\pi m}{\mathfrak{N}} - z\sqrt{\frac{\pi}{2\mathfrak{N}}}; \frac{i}{\mathfrak{N}}] \quad (24)$$

We denote as $\mathfrak{z}_n$ the zeros of function $f(z)$ in Eq.(23) and we denote as η_n the zeros of function

$$g(z) = \pi^{-1/4} \sum_{m=0}^{\mathfrak{N}-1} \mathfrak{U}_{ml} \mathbb{F}_l\ \vartheta_3[\frac{\pi m}{\mathfrak{N}} - z\sqrt{\frac{\pi}{2\mathfrak{N}}}; \frac{i}{\mathfrak{N}}] \quad (25)$$

The paths of the zeros define completely a finite quantum system. Hence the study of paths of the zeros is equivalent the study of the system. We consider the paths of the zeros of both functions $f(z)$ and $g(z)$.
Let $\mathfrak{z}_0(t), \mathfrak{z}_1(t), \mathfrak{z}_2(t)$ be the paths of the three zeros of $f(z)$, and let $\eta_0(t), \eta_1(t), \eta_2(t)$ be the paths of the three zeros of $g(z)$.

Example 4:
We consider the Hamiltonian

$$H = \left[\begin{array}{ccc} 1 & -i & 0 \\ i & 1 & 0 \\ 0 & 0 & 2 \end{array}\right] \quad (26)$$

which has the eigenvalues $0, 2, 2$ with period $\mathcal{O} = \pi$. We calculate the Hamiltonian $\mathfrak{U} H \mathfrak{U}^\dagger$ which has the same eigenvalues of H. We assume that the initial values the zeros of $f(z)$ are

$$\mathfrak{z}_0(0) = 2 + 2i,\ \mathfrak{z}_1(0) = 2.2 + 2i,\ \mathfrak{z}_2(0) = 2.3 + 2.3i \quad (27)$$

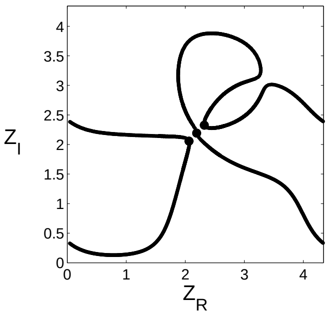

Fig. 5. The path of the zeros $\mathfrak{z}_0(t)$, $\mathfrak{z}_1(t)$, $\mathfrak{z}_2(t)$ for the system with the Hamiltonian of Eq.(26). All zeros follow the same path. At $t = 0$ the zeros $\mathfrak{z}_0(0)$, $\mathfrak{z}_1(0)$, $\mathfrak{z}_2(0)$ are given in Eq.(27)

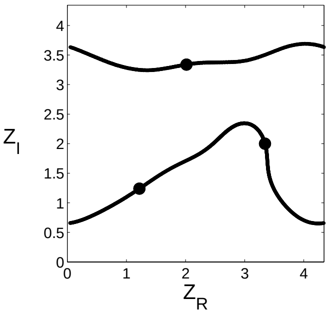

Fig. 6. The path of the zeros $\eta_0(t)$, $\eta_1(t)$, $\eta_2(t)$ for the system with the Hamiltonian $\mathfrak{U}H\mathfrak{U}^\dagger$ where H of Eq.(26). The zeros $\eta_0(t)$, $\eta_2(t)$ follow the same path and the zero $\eta_1(t)$ follows a different path.. At $t = 0$ the zeros $\eta_0(0)$, $\eta_1(0)$, $\eta_2(0)$ are given in Eq.(28) .

The initial values the zeros of $g(z)$ are

$$\mathfrak{z}_0(0) = 1 + 1i,\ \mathfrak{z}_1(0) = 2 + 3.3i,\ \mathfrak{z}_2(0) = 3.4 + 1i \quad (28)$$

In the case of Eq.(27) we get

$$\mathfrak{z}_0(T + t) = \mathfrak{z}_1(t),\ \mathfrak{z}_1(T + t) = \mathfrak{z}_2(t),\ \mathfrak{z}_2(T + t) = \mathfrak{z}_0(t) \quad (29)$$

After period the three zeros follow the same path. In Fig.5 we present the paths of these zeros.
In the case of Eq.(28), after period we found numerically that

$$\mathfrak{z}_0(\mathcal{O} + t) = \mathfrak{z}_2(t),\ \mathfrak{z}_2(\mathcal{O} + t) = \mathfrak{z}_0(t). \quad (30)$$

Therefore two of the zeros follow the same path and the third one follows a different path. In Fig.6. we present the paths of these zeros.

Example 5:
Another example is the Hamiltonian of Eq.(26) and zeros with the initial values

$$\mathfrak{z}_0(0) = 1.4 + 3.4i,\ \mathfrak{z}_1(0) = 1.7 + 2.5i,\ \mathfrak{z}_2(0) = 3.4 + 0.6i \quad (31)$$

and the initial values of zeros of $g(z)$ are

$$\mathfrak{z}_0(0) = 0.8 + 3.9i,\ \mathfrak{z}_1(0) = 2 + 0.36i,\ \mathfrak{z}_2(0) = 3.7 + 2.3i, \quad (32)$$

The period is $\mathcal{O} = \pi$.
In the case of Eq.(31) after period the zeros obey the relation

$$\mathfrak{z}_0(\mathcal{O} + t) = \mathfrak{z}_2(t),\ \mathfrak{z}_2(\mathcal{O} + t) = \mathfrak{z}_1(t),\ \mathfrak{z}_1(\mathcal{O} + t) = \mathfrak{z}_0(t) \quad (33)$$

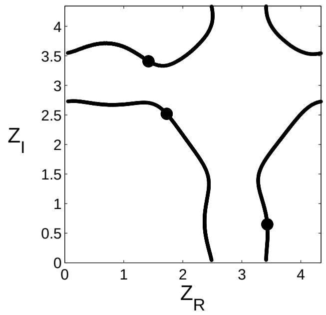

Fig. 7. The path of the zeros $\mathfrak{z}_0(t)$, $\mathfrak{z}_1(t)$, $\mathfrak{z}_2(t)$ for the system with the Hamiltonian of Eq.(26). All zeros follow the same path. At $t = 0$ the zeros $\mathfrak{z}_0(0)$, $\mathfrak{z}_1(0)$, $\mathfrak{z}_2(0)$ are given in Eq.(31)

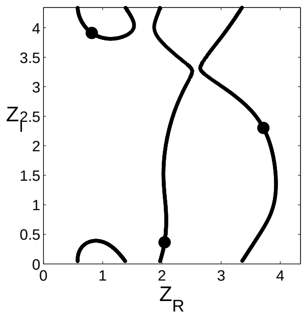

Fig. 8. The path of the zeros $\eta_0(t)$, $\eta_1(t)$, $\eta_2(t)$ for the system with the Hamiltonian $\mathfrak{U}H\mathfrak{U}^\dagger$ where H of Eq.(26). The zeros $\eta_0(t)$, $\eta_1(t)$, $\eta_2(t)$ follow a different paths.. At $t = 0$ the zeros $\eta_0(0)$, $\eta_1(0)$, $\eta_2(0)$ are given in Eq.(32)

Here the three zeros follow the same path. In Fig.7. we present the paths of these zeros.
In the case of Eq.(32) found numerically that each zero follows a different path.
In Fig.8. we present the paths of these zeros.
A unitary transformation is equivalent to a change of basis. As the basis change the quantum state $|f\rangle$ transforms into different quantum state.

Let U be an arbitrary unitary transformation . We can define a map from torus T_1 into another torus T_2

$$G : T_1 \longrightarrow T_2 \quad (34)$$

as following

$$G(f(z)) = g(z) = \pi^{-1/4} \sum_{m=0}^{\mathfrak{N}-1} U_{ml} \mathbb{F}_l\ \vartheta_3[\frac{\pi m}{\mathfrak{N}} - z\sqrt{\frac{\pi}{2\mathfrak{N}}}; \frac{i}{\mathfrak{N}}] \quad (35)$$

where $f(z)$ is the analytic function in Eq.(23). It seen that this map is one-to-one and on to.

Let us try define another map from torus T_1 into T_2

$$W : T_1 \longrightarrow T_2 \quad (36)$$

such that

$$W(\mathfrak{z}_n) = \eta_n \quad (37)$$

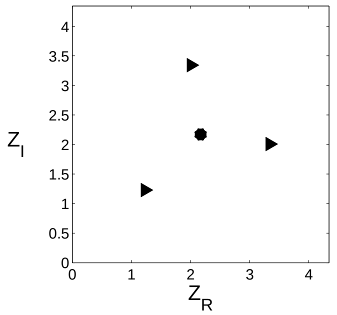

Fig. 9. The zero of $f(z)$(circle). At $t = 0$ the zeros $\mathfrak{z}_0(0)$, $\mathfrak{z}_1(0)$, $\mathfrak{z}_2(0)$ are given in Eq.(38) .The zeros of $g(z)$(triangles). At $t = 0$ the zeros $\eta_0(0)$, $\eta_1(0)$, $\eta_2(0)$ are given in Eq.(39)

Where $\mathfrak{z}_n$ are the zeros of $f(z)$ and η_n are the zeros of $g(z)$. The domain of this map is the zeros of function $f(z)$ and the range is the zeros of function $g(z)$. This map is not one-to-one, it is enough to give the following example to show that. Let $U = \mathfrak{U}(1, -1, -1)$ in Eq.(22). We assume that the initial zeros of $f(z)$ are

$$\mathfrak{z}_0(0) = \mathfrak{z}_1(0) = \mathfrak{z}_2(0) = 2.1708 + 2.1708i. \qquad (38)$$

In this case the three zeros are identical, we can say that they are one zero.

The initial values of the zeros of $g(z)$ are

$$\mathfrak{z}_0(0) = 1 + 1i,\ \mathfrak{z}_1(0) = 2 + 3.34i,\ \mathfrak{z}_2(0) = 3.34 + 2i \quad (39)$$

In Fig.9 we present the zeros of $f(z)$ (circles), and the zeros of $g(z)$ (triangles).

Problem 2

Let $\mathfrak{z}_n$ be the zeros of the analytic function $f(z)$ in Eq.(23) and η_n be the zeros of the analytic function $g(z)$ in Eq.(25). Let

$$W : T_1 \longrightarrow T_2; \qquad (40)$$

be a map from torus T_1 into another T_2 such that

$$W(\mathfrak{z}_n) = \eta_n. \qquad (41)$$

What is the definition of that map?

V. Conclusion

We discussed briefly the analytic representation of finite quantum systems. We reviewed briefly the zeros of analytic theta function and there time evolution.

We showed that in some cases $\mathfrak{N}$ of the zeros follow the same path and in other cases each zero follow its own path. Numerically we found that the same zeros with two different Hamiltonian, in the first case follow the same path and in the second case follow different paths. Also we showed that for the same Hamiltonian, two sets of zeros, follow the same path and different paths correspondingly. We concluded that there is specific constraint should be satisfied, and either the zeros or the Hamiltonian subjected to the constraint which should involve both the zeros and the Hamiltonian. The first problem is to construct the constraint. A unitary transformation is equivalent to a change of basis. We try to discuss how to define a map from torus to another such that the domain and the range is the zeros of analytic functions. The second problem is to construct the map. We gave several examples to demonstrate these ideas.

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