Joint Microstatistic Multiuser Detection and Cancellation of Nonlinear Distortion Effects for the Uplink of MC-CDMA Systems Using Golay Codes

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Abstract—The study in this paper underlines the importance of correct joint selection of the spreading codes for uplink of multicarrier code division multiple access (MC-CDMA) at the transmitter side and detector at the receiver side in the presence of nonlinear distortion due to high power amplifier (HPA). The bit error rate (BER) of system for different spreading sequences (Walsh code, Gold code, orthogonal Gold code, Golay code and Zadoff-Chu code) and different kinds of receivers (minimum mean-square error receiver (MMSE-MUD) and microstatistic multi-user receiver (MSF-MUD)) is compared by means of simulations for MC-CDMA transmission system. Finally, the results of analysis will show, that the application of MSF-MUD in combination with Golay codes can outperform significantly the other tested spreading codes and receivers for all mostly used models of HPA.

Keywords—HPA, MC-CDMA, microstatistic filter, multi-user receivers, PAPR.

I. INTRODUCTION

M C-CDMA is a modulation technique that exploits the advantages of spread spectrum and orthogonal frequency division multiplexing (OFDM) [1]. Inherent benefits of both OFDM and spread spectrum technique include robustness against multipath fading, very high spectrum efficiency and the ability to accommodate large number of active users without decreasing the performance of transmission system considerably. However the bit error performance degrades very rapidly if orthogonality between subcarriers and in addition between spreading codes of active users can not be maintained. Reasons for this include selective fadings, frequency offsets, delay spread exceeding the guard interval and eventually nonlinear distortions.

Throughout this paper, high sensitivity to nonlinear amplification of MC-CDMA transmission system is investigated. Particularly for MC-CDMA, where orthogonal spreading codes are used to reduce the multiple access interference (MAI), the non-linearity (NL) destroys the orthogonality among spreading codes thus increasing MAI. The conventional solution to this problem is to back-off the operating point of nonlinear amplifier and, as a consequence, inefficient use of power amplifiers. On the other hand, using low back-offs leads to signal distortion and, as a result, increased performance degradation. Alternative techniques can be found in the literature to reduce this effect. Frequently used solutions at the transmitter

Authors are with the Department of Electronics and Multimedia Communications, Faculty of Electrical Engineering and Informatics, Technical University of Košice, Slovak Republic). side include predistortion, active constellation extension, tone reservation, selected mapping [2] and different code allocation strategies [3]. Strategies at the receiver side usually combine iterative decoding and multi-user detection in such a way that both HPA non-linearity compensation and multiple access interference (MAI) are taken into account [4], [5], [6].

Recently in [3],[7], different spreading sequences for MC-CDMA systems have been compared from MAI and peakto-average power ratio (PAPR) perspective. Nevertheless, the aim of this contribution is to analyse the joint impact of PAPR, MAI and nonlinear distortion due to HPA from the BER point of view. As it will be presented, overall system performance can be significantly improved by joint using Golay sequences in the MC-CDMA system and nonlinear detector capable to better approximate nonlinear borders of receiver decision regions at the receiver side, respectively. We will show that this method has superior performance from all assumed models of HPA.

The rest of the paper is organised as follows. In the next section short overview of nonlinear effects due to HPA is presented. The basis of microstatistic filtering and its application to MUD are given in Section 3. In the Section 4 MC-CDMA system model and configurations are described. In following Section the performance properties of uplink MC-CDMA transmission systems in the presence of HPA non-linearities will be studied and analysed by using a set of computer simulations. Within these simulations, it will be shown, that new proposed combination of applying Golay sequences for signal spreading at transmitter of MC-CDMA system and microstatistic MUD at the receiver side outperforms very clearly other combinations of spreading sequences and receivers. Finally, conclusions and final remarks to this contribution are drawn in Section 6.

II. NONLINEAR EFFECTS OF HPA IN MC-CDMA SYSTEM

Multi-carrier schemes such as MC-CDMA are known to suffer from large envelope fluctuation of the transmitted signal, caused by the addition of a large number of independently modulated subcarriers. The output of multi-carrier modulation based on IFFT operation is usually characterized by high PAPR, which can be defined as [2]

$$PAPR = \frac{max|s(t)|^2}{E[|s(t)|^2]}$$
(1)

where E[.] is the expectation operator. It can be shown (e.g. [7]), that in the case of MC-CDMA systems, PAPR

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TABLE I PAPR BOUNDS OF MC-CDMA UPLINK SIGNAL

Spreading code	PAPR(upper bound)
Walsh	$\leq 2N_c$
Gold, Orthogonal Gold	$\leq 2(t(m) - 1 - \frac{t(m)+2}{L})$
Golay	≤ 4
Zadoff-Chu	2

parameter strongly depends on the applied spreading sequence. The PAPR upper bound of different spreading sequences is summarised in Table I [7]. As it can be seen the PAPR bound for Golay codes and Zadoff-Chu codes is independent of the spreading code length L. When number of subcarriers N_c is a multiple of L, as in our analysis, the PAPR of the Walsh-Hadamard code is upper-bounded by $2N_c$ [1].

Because of the large envelope fluctuations of the input signal of HPA, the real HPAs have to be modelled as nonlinear amplifiers. The non-linear HPA can be modelled as a memoryless device. Let $x(t) = u_x e^{-j\phi(t)}$ be the HPA complex input signal with amplitude u_x and phase $\phi(t)$. The corresponding output signal can be written as

$$y(t) = G_{u_x} e^{-j\Phi(t)} \tag{2}$$

where $G_{u_x} = f(u_x)$ describes the amplitude-to-amplitude (AM/AM) conversion. The amplitude-to-phase (AM/PM) distortion $\Phi(t) = g(u_x)$ produces an additional phase modulation [8].

In this paper, we compare nonlinear effects due to HPA modelled by Saleh, Ghorbani and Rapp models. Rapp and Ghorbani models are used to model solid state power amplifiers (SSPA). Whereas the Saleh model is used to model traveling wave tube amplifiers (TWTA).

In the case of Saleh model of HPA, AM/AM and AM/PM characteristics are given by [8]

$$G_{u_x} = \frac{\kappa_G u_x}{1 + \chi_G u_x^2} \tag{3}$$

$$\Phi_{u_x} = \frac{\kappa_\Phi u_x^2}{1 + \chi_\Phi u_x^2}.$$
(4)

For the precise specification of the Saleh model of HPA, the parameters $\kappa_G = 2$, $\chi_G = \chi_{\Phi} = 1$ and $\kappa_{\Phi} = \pi/3$ have been chosen for simulations. On the other hand, AM/AM and AM/PM characteristics of Ghorbani model are as follows [9]

$$G_{u_x} = \kappa_{G_1} u_x + \frac{\kappa_{G_2} u_x^{\varepsilon_G}}{1 + \chi_G u_x^{\varepsilon_G}}$$
(5)

$$\Phi_{u_x} = \kappa_{\Phi_1} u_x + \frac{\kappa_{\Phi_2} u_x^{\varepsilon_{\Phi}}}{1 + \chi_{\Phi} u_x^{\varepsilon_{\Phi}}}.$$
 (6)

In case of Ghorbani model of HPA, its parameters have been set to $\kappa_{G_1} = 0$, $\kappa_{G_2} = \chi_G = 20$, $\varepsilon_G = \varepsilon_{\Phi} = 2$, $\kappa_{\Phi_1} = \pi/15$, $\kappa_{\Phi_2} = \pi/3$ a $\chi_{\Phi} = \pi$.

And finally, Rapp model of HPA is described by the following AM/AM and AM/PM characteristics [10]

$$G_{u_x} = \frac{\kappa_G u_x}{(1 + (\frac{u_x}{O_{sat}})^{2s})^{\frac{1}{2s}}}$$
(7)

$$\Phi_{u_x} = 0 \tag{8}$$

where s = 3 is the smoothness factor and $O_{sat} = 1$ is the output saturation level.

The HPA operation in the region of its nonlinear characteristic causes a nonlinear distortion of transmitted signal, that subsequently results in higher BER and out-ofband energy radiation. The operating point of HPA is defined by parameter input back-off (*IBO*) which corresponds to the ratio of saturated input power (P_{max}) and average input power ($\overline{P_x}$) [6]

$$IBO_{dB} = 10\log_{10}(\frac{P_{max}}{\overline{P_x}}).$$
(9)

The measure of effects due to nonlinear HPA could be decreased by the selection of relatively high value of *IBO*. However, this approach will result in inefficient use of HPA performance resulting in decreasing radius of the area covered by the effective MC-CDMA signal. Another solutions which include decreasing of nonlinear HPA influence are represented by a number of methods implemented in transmitter or/and receiver side of MC-CDMA transmission system [2], [3]. It has been proposed in [6] that MC-CDMA performance improvement under condition of nonlinear amplification could be reached also by the application of nonlinear receivers.

It follows from above mentioned facts that the MC-CDMA performance expressed by BER strongly depends on spreading sequences, HPA model and the applied receiver. The spreading sequences selection depends on the PAPR of MC-CDMA signal as well as on the level of MAI due to cross-correlation properties of the particular spreading sequences. Because of non-linear distortion due to HPA and MAI inherency, it is expected that the application of nonlinear and multi-user receiver will overcome linear receiver.

III. MICROSTATISTIC FILTERING BASED MUD

The structure of MSF MUD is very similar to that of conventional linear MMSE-MUD. In fact, MSF-MUD can be obtained from linear MMSE-MUD by just substituting set of multi-channel linear filters in MMSE-MUD, located between BMF and demmaper devices, by the complex-valued multi-channel conventional microstatistic filter (C-M-CMF). A block scheme of C-M-CMF is given in Fig. 1 [11], where M, y(i) and $\hat{d}(k)$ denote the number of the input and output channels, the *i*-th input complex signal and the *k*-th output complex signal of the C-M-CMF, respectively. As it can be observed the C-M-CMF consists of M complex-valued threshold decomposers (TD) and a set of M multi-channel Wiener filters (WF).

The *i*-th TD, denoted as TD_i , performs a threshold decomposition of the signal y(i) into a set of the *L* complexvalued signals y(i, j). The outputs of each TD are uniquely determined from its input signal by

$$\mathbf{Y}(i) = [y(i,1), \cdots, y(i,L)]^{\mathsf{T}} = \mathbf{D}_{i} \{Y(i)\} e^{j\phi(i)} = = [Y(i,1), \cdots, Y(i,L)]^{\mathsf{T}} e^{j\phi(i)}$$
(10)

where Y(i) = |y(i)|, $\phi_i = \arg \{y(i)\}$ and the superscript T signifies matrix transposition. The term $D_i\{Y(i)\}$ represents



Fig. 1. Complex-valued multi-channel conventional microstatistic filter

the threshold decomposition of Y(i) into a set of L signals as

$$Y(i,j) = \begin{cases} 0 & \text{if } Y(i) < l(i,j-1) \\ Y(i) - l(i,j-1) & \text{if } l(i,j-1) \le Y(i) < \\ & < l(i,j) \\ l(i,j) - l(i,j-1) & \text{if } l(i,j) \le Y(i) \end{cases}$$
(11)

for $1 \leq j \leq L$. The parameters l(i, j) constituting the vector $\mathbf{L}_i = [l(i, 1), \dots, l(i, L)]^{\mathsf{T}}$ are the positive real-valued threshold levels of TD_i , which satisfy $0 = l(i, 0) < l(i, 1) < \dots < l(i, L) = \infty$ [6].

According to Fig. 1, the output signals obtained after threshold decomposition are fed into the set of M multichannel WFs. Let WF_k, k = 1, 2, ..., M denote the k-th WF, then the k-th output of C-M-CMF is computed as

$$\hat{d}(k) = \mathbf{H}^{\mathsf{H}}(k)\mathbf{Y} \tag{12}$$

where $\mathbf{H}(k) = [h(k,0), \mathbf{H}'^{\mathsf{T}}(k)]^{\mathsf{T}}, \mathbf{Y} = [1, \mathbf{Y}'^{\mathsf{T}}]^{\mathsf{T}}$ and the superscript H denotes Hermitian transposition. \mathbf{Y}' and $\mathbf{H}'(k)$ are two vectors of the same length that comprise the output signals from all TDs and the coefficients of k-th WF, respectively. The constant term h(k,0) is applied to the C-M-CMF structure in order to obtain an unbiased estimation of desired signal at the output of WF_k.

Before being able to use the microstatistic filter one has to determine the threshold levels and the coefficients of the WFs. This is done by transmitting a special training sequence and by following the MMSE criterion. For the sake of brevity we do not introduce this procedure in this contribution. Detailed description can be found in [6], [11].

IV. SYSTEM CONFIGURATION OF MC-CDMA

The basic scenario of introduced study of nonlinear effects in MC-CDMA systems is represented by uplink of MC-CDMA transmission system at 16-QAM baseband modulation, for 25% user load. In this paper, only AWGN channel is considered because the aim of this contribution is to concentrate on the analysis of non-linearity effects.



Fig. 2. MC-CDMA transmitter



Fig. 3. MC-CDMA receiver

A block diagram of the simplified baseband model of MC-CDMA transmitter is given in the Fig. 2. It can be seen from this figure, that the information bits to be transmitted by a particular user are firstly fed to the block of non-recursive, non-systematic convolution encoder with rate R = 1/3. Then, encoded bits are baseband modulated into 16-QAM modulation symbols and then they are spread by using a specific spreading sequence c_m . As the spreading sequences, Walsh codes, Gold codes and orthogonal Gold codes with period of L = 32 chips as well as complementary Golay codes and polyphase Zadoff-Chu codes with period of L = 31chips are applied. The spread symbols are modulated by multi-carrier modulation implemented by inverse fast Fourier transformation operation (IFFT). In our simulations, the total number of sub-carriers has been set to $N_c = 128$. In order to avoid aliasing and the out-of-band radiation into the data bearing tones, the oversampling rate of 4 has been applied. The IFFT block outputs after parallel-to-serial conversion represent the input signal of HPA. In presented paper, Saleh, Rapp and Ghorbani model of HPA is assumed. In the transmitter model according to Fig. 2, the traditional block of cyclic prefix insertion is not included due to AWGN channel assumption.

The basic structure of receivers considered in this paper is sketched in the Fig. 3. It can be seen from this figure, that the receiver consists of serial-to-parallel converter (S/P), blocks of fast Fourier transformation (FFT), bank of matched filters (BMF), block of linear or non-linear transformation (labelled as T) and the M-QAM demapper block and finally block of convolution decoding. Here, the operation of a single-user receiver known as BMF consists of set of simple matched filters (correlators). A deeper description of MC-CDMA transmission systems can be found e.g. in [12] or [13].

In order to extend BMF into multi-user receiver, the Ttransformation block is included in the receiver structure. In this paper, the linear MMSE-MUD [12] as well as nonlin-



Fig. 4. *BER* vs. *IBO* for MC-CDMA transmission system for different spreading sequences. HPA model: Saleh model. $E_b/N_0 = 17 dB$

ear MSF-MUD for MC-CDMA [6], [11] are considered. Ttransformation block in MMSE-MUD is represented by multichannel linear Wiener filter. In the case of MSF-MUD, the T-transformation block is represented by C-M-CMF as it was described in previous section.

V. MC-CDMA PERFORMANCE ANALYSIS

MC-CDMA performance analysis presented in this section is based on computer simulations.

A. Saleh Model

Firstly, let us assume the Saleh model of HPA for different spreading codes and different receivers.

In the Fig. 4, *BER vs. IBO* for $E_b/N_0 = 17 \ dB$ is illustrated. It can be seen from this figure, that the worst performance of MC-CDMA is provided by the Walsh code application due to the high *PAPR* of MC-CDMA signal resulting in severe nonlinear distortion and loss of orthogonality. On the other hand, the best performance can be provided by the application of Golay codes in combination with MSF-MUD. This performance can be explained by relatively low *PAPR* of MC-CDMA signal, when Golay codes are employed and the MSF-MUD ability to compensate nonlinear distortion due to HPA. It follows from this figure, that there is the interval of lower *IBO* values for which the application of MSF-MUD can provide meaningful improvement of the MC-CDMA performance in comparison with that of MMSE-MUD application.

The performance of the MC-CDMA transmission system expressed by *BER vs.* E_b/N_0 for *IBO* = 0 *dB*, *IBO* = 3 *dB* is illustrated by Fig. 5 and 6. These simulation results confirms the previous conclusions i.e. good bit error performance can be obtained by using Golay codes in combination with MSF-MUD. On the contrary, the other investigated codes show insufficient results, that can be explained by higher *PAPR* of MC-CDMA signal when these codes are adopted



Fig. 5. BER vs. E_b/N_0 for MC-CDMA transmission system for different spreading sequences. HPA model: Saleh model. IBO = 0dB



Fig. 6. BER vs. E_b/N_0 for MC-CDMA transmission system for different spreading sequences. HPA model: Saleh model. IBO=3dB

for spreading. Note, that in these cases it is highly recommended to use additional method for PAPR reduction at the transmitter side in order to suppress this undesirable effect and to get improved bit error performance.

B. Ghorbani Model

In this section the performance of MC-CDMA transmission system at Ghorbani model of HPA is discussed.

As can be seen from Fig. 7 the great performance gain is achieved again with combination of MSF-MUD at the receiver side and Golay codes application at the transmitter side.

In the Fig. 8 and Fig. 9, $BER vs. E_b/N_0$ for IBO = 0 dB and IBO = 3 dB for Ghorbani Model of HPA is illustrated. As it was in the previous cases the best performance can be provided by the application of Golay codes in combination with MSF-MUD. On the other hand the other combinations of spreading codes and receiver fail and provide poor performance results particularly for very low *IBO*. Especially



Fig. 7. BER vs. IBO for MC-CDMA transmission system for different spreading sequences. HPA model: Ghorbani model. $E_b/N_0=17dB$



Fig. 8. BER vs. E_b/N_0 for MC-CDMA transmission system for different spreading sequences. HPA model: Ghorbani model. IBO=0dB

for IBO = 0dB the application of MSF-MUD can provide meaningful improvement of the MC-CDMA performance in comparison with that of MMSE-MUD application.

C. Rapp Model

In this section Rapp model of HPA is assumed in our simulations.

Fig. 10 demonstrates $BER vs. E_b/N_0$ for MC-CDMA transmission system for IBO = 0 dB. From this figure it can be observed that in the case of Rapp model of HPA all codes provide almost the same results and the performance of MSF-MUD is almost identical to that of MMSE-MUD. This results from fact, that MSF-MUD as an nonlinear receiver can provide performance gain over the use of linear MMSE receiver when nonlinear distortion of signal is more severe. If distortion caused by HPA is smaller or closer to linear model of HPA the MSF-MUD can not offer significant performance improvement. It can be concluded that distortion caused by



Fig. 9. BER vs. E_b/N_0 for MC-CDMA transmission system for different spreading sequences. HPA model: Ghorbani model. IBO=3dB



Fig. 10. BER vs. E_b/N_0 for MC-CDMA transmission system for different spreading sequences. HPA model: Rapp model. IBO = 0dB

Rapp model of HPA is smaller than that of Saleh model or Ghorbani model of HPA.

VI. CONCLUSION

In this paper, we have investigated possibility of suppression of nonlinear effects due to HPA by joint application of Golay codes for spreading at the transmitter, resulting in lower PAPR of MC-CDMA signal, and MSF-MUD based multiuser detection, capable to better approximate nonlinear borders of receiver decision regions at the receiver side. We have analysed performance of MC-CDMA transmission system for three frequently used models of HPA (Saleh, Ghorbani and Rapp model), the different spreading sequences (Walsh, Golay, Gold, orthogonal Gold and Zadoff-Chu), *IBO* parameter values and receiver types (MMSE-MUD, MSF-MUD). It has been found that proposed application of the Golay codes as the spreading sequences in combination with MSF-MUD can overcome clearly the other investigated spreading sequences and receivers for all assumed HPA models, especially for a certain interval of the *IBO* values. It should be emphasised that MSF-MUD provides always better or at least the same performance than that of MMSE-MUD.

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