

Analysis of explosive shock wave and its application in snow avalanche release

Mahmoud Zarrini and R.N. Pralhad

Abstract—Avalanche velocity (from start to track zone) has been estimated in the present model for an avalanche which is triggered artificially by an explosive device. The initial development of the model has been from the concept of micro-continuum theories [1], underwater explosions [2] and from fracture mechanics [3] with appropriate changes to the present model. The model has been computed for different slab depth R , slope angle θ , snow density ρ , viscosity μ , eddy viscosity η^* and couple stress parameter η . The applicability of the present model in the avalanche forecasting has been highlighted.

Keywords—Snow Avalanche velocity, Avalanche zones, Shock wave, Couple stress fluids.

I. INTRODUCTION

AVALANCHES are sudden downward movement of snow in the hilly regions. It is estimated that several million dollars property damage due to avalanches during the winter period. In addition to the property damage, lots of human lives getting lost have also been observed in the avalanches. In view of its unpredictable nature of release and losses to human lives and property, avalanches during the modern days are being triggered artificially by explosive devices. It is understood that shocks created as a result of detonation in the snow pack may induce pressure which will overcome the yield value of the material (snow) and sets into the downward motion yielding an avalanche ([4]-[7]).

Modeling of explosive shock waves has been undertaken with a view to estimate shock velocity in the snow pack and its propagation down the slope hill. It is amused that, flow of an avalanche is similar to that of hydrodynamic behavior and basic momentum equations are represented by Navier-Stokes equation [8] coupled with body forces from solid mechanics ([9]-[11]) aspect. In addition to the above, snow mass has been treated as non-continuum approach and effects due to couple stresses ([1], [12] and [13]) have also been accounted while modeling the proposed study.

II. ANALYSIS

Momentum equation in direction of axis for avalanche release by considering the snow movement down the hill in two different zones (starting zone and track zone) (see fig. 1) with couple stress effects [1] can be written as;

$$\eta \nabla^4 V + \mu \nabla^2 V + \rho \frac{DV}{Dt} + \nabla P - F = 0 \quad (1)$$

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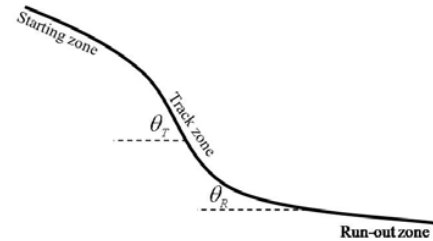


Fig. 1. Three stages of an avalanche release

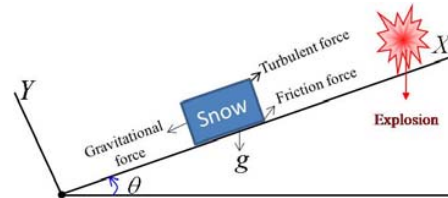


Fig. 2. Body force representation on a slab and Explosion at start zone

Here μ snow viscosity, ρ snow density, V represents avalanche velocity (in three dimensional), p represents pressure and F is body force. Assuming the flow is steady, one dimensional and pressure is that of atmospheric and constant, Equation (1) simplifies to:

$$\eta \frac{\partial^4 u}{\partial x^4} + \mu \frac{\partial^2 u}{\partial x^2} + \rho u \frac{\partial u}{\partial x} - F = 0 \quad (2)$$

Body force F is taken in the form of (See Fig.2):

$$F = \rho g (\sin \theta - \mu \cos \theta) - \frac{\rho g}{R\eta^*} u^2 \quad (3)$$

Where u is axial velocity, θ is slope angle, x is distance down the hill (start to track zone), η^* is eddy viscosity and R is slab depth. Then equation (2) will be:

$$\eta \frac{\partial^4 u}{\partial x^4} + \mu \frac{\partial^2 u}{\partial x^2} + \rho u \frac{\partial u}{\partial x} + \frac{\rho g}{R\eta^*} u^2 - \rho g (\sin \theta - \mu \cos \theta) = 0 \quad (4)$$

We need four conditions to solve Equation (4). Following initial conditions are assumed in the present model.

1. $u|_{x=x_s} = 0.0029 \text{ m/s}$
2. $\frac{\partial u}{\partial x}|_{x=x_s} = 345 \text{ s}^{-1}$
3. $\frac{\partial^2 u}{\partial x^2}|_{x=x_s} = \frac{\omega_0}{L(1+\bar{\eta})}$
4. $\frac{\partial^3 u}{\partial x^3}|_{x=x_s} = 0$

Where $\bar{\eta} = \frac{\eta}{\eta^*}$ is the non-dimensional couple stress parameter and $\dot{\eta}$ is gradient of the viscosity, x_s is starting zone. The

data required for the computation is taken from Refs. ([2], [11], [14] and [15]) and are tabulated in Table 1.

TABLE I
DATA OF FLOW PARAMETERS

Symbol	Quantity	Range	Typical Value
θ	Slope angle	30 – 45	38°
η^*	Eddy viscosity	400 – 1000	600 m/s ²
ρ	Density of snow	100 – 300	200 kg/m ³
R	Slab depth	0.4 – 1	0.5 m
L	Slab length	1 – 3	2 m
μ	Viscosity of snow	0.1 – 0.3	0.2 kg/m.s
ω_0	Angular velocity	1 – 7	4 s ⁻¹
η	Couple stress parameter	0 – 1	0, 0.5 kg/m.s
P_m	Peck pressure	200 – 300	300 pa
c	Sound velocity in snow	100 – 300	300 m/s
T	Time decay	1 – 15	1 s

Initial conditions 1 and 2 mentioned above are obtained by the method suggested in Refs. ([15],[16]). The brief is mentioned below. Equation of momentum for the explosive shocks of the material getting detonated at the top hill covered with snow is given by [15]

$$2 P(t) - \rho c V(t) = m \frac{dV}{dt} \quad (5)$$

Here $P(t)$ is the shock pressure (as a result of an explosion), c is the sound velocity in snow, V is the velocity of snow pack (in fracture zone) and m is snow mass per unit area [15]

$$P(t) = P_m \exp\left(-\frac{t}{T}\right) \quad (6)$$

P_m , t and T are respectively peak pressure, time and time decay of detonation. Using condition $V(0) = 0$, equation (5) can be analytically solved for:

$$V(t) = \frac{2 P_m}{c \rho (z - 1)} \left(\exp\left(-\frac{t}{z T}\right) - \exp\left(-\frac{t}{T}\right) \right) \quad (7)$$

Where

$$z = \frac{m}{c \rho T} = \frac{L}{c T}$$

Maximum velocity V_m can be estimated by taking $\frac{dV}{dt} = 0$:

$$\frac{dV}{dt} = 0, \text{ so}$$

$$t_m = T z \ln \left(\frac{z}{z - 1} \right) \quad (8)$$

Substituting eqn. (8) in eqn. (7):

$$V_m = \frac{2 P_m}{c \rho} z^{\frac{z}{1-z}} \quad (9)$$

The peak velocity (V_m) which is obtained by the method described above has been used as an initial data (or condition) for the computation of momentum equation (4). It is estimated that $u(0) = 0.0029 \text{ m/s}$ [$V_m(t_m)$] and $\dot{u}(0) = 345 \text{ s}^{-1}$ [$\frac{V_m}{t_m}$] for the present computation aspect. Having known all the four conditions, Equation (4) has computed for the two aspects [with couple stress effects ($\eta \neq 0$) and for without couple stress effects ($\eta = 0$)].

A. Avalanche dynamic pressure

In addition to the axial velocity computation, present model has also been computed for dynamic pressure by using the relation

$$p = \frac{1}{2} \rho u^2 \quad (10)$$

III. RESULTS AND DISCUSSIONS

1. If $\eta = 0$ then, only two conditions are sufficient to solve the following equation:

$$\mu \frac{\partial^2 u}{\partial x^2} + \rho u \frac{\partial u}{\partial x} + \frac{\rho g}{R \eta^*} u^2 - \rho g (\sin \theta - \mu \cos \theta) = 0 \quad (11)$$

Equation (11) has been solved by the method of Runge - Kutta Fourth Order method. Use of MATLAB Software has been made use while solving this equation. The computed results have been shown in Figs. 3 to 7.

2. If $\eta \neq 0$ then, Equation (4) has been computed again by the Runge - Kutta Fourth Order method and the computed results have been shown in Figs. 8 to 13.

The results indicate that the snow avalanche velocity increases with the increase in eddy viscosity (η^*), slab depth (R) and slope angle (θ), however it decreases with the increases in viscosity (μ). The effect of density (ρ) on the avalanche velocity has observed no effect on the same which is strange. For the case of the results are found to be the same pattern except for the case of density (ρ). The density has found profound effect on the flow where higher the density of snow yield higher avalanche velocity (between start zone and track zone) which agrees with the physical observations. In all the computed results, it is observed that the computed results for the case of without couple stress $\eta = 0$ are higher to that compared to the case of with couple stress $\eta \neq 0$. These results of higher values are in agreement with the physical observation since effects of a case without couple stresses will not resist the motion down the hill whereas couple stresses do resist the motion and hence lower in values.

Dynamic pressure which is one of the required parameter in the design of control structure has been computed by using Eqn. (10). The results are shown in Fig. (14). The results indicate that, avalanche dynamic pressure increases with decreasing couple stress parameters. Artificially released avalanches induce less dynamic pressure when compared to that of naturally released one [14].

IV. CONCLUSION

Applications of Explosive shock waves have been made use of for the analysis of artificial release of an avalanche in the present studies. The model has been developed with a view to account for couple stresses and its effects on various flow parameters such as eddy viscosity (η^*), density (ρ), slope angle (θ), slab depth (R) and couple stress parameter (η). The model has also been compared to that of naturally released

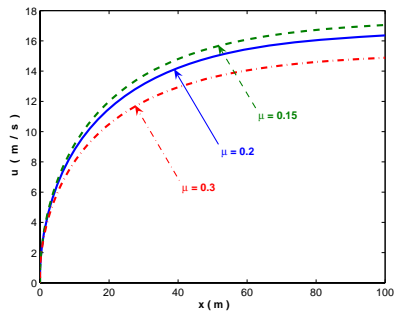


Fig. 3. Variation of velocity with x for different Snow Viscosity μ

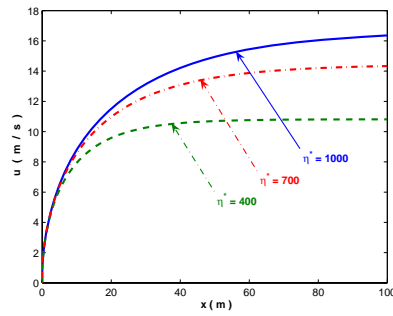


Fig. 7. Variation of velocity with x for different Eddy viscosity η^*

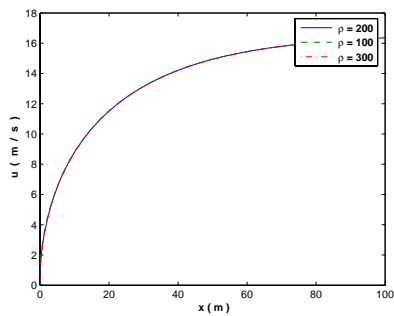


Fig. 4. Variation of velocity with x for different Snow density ρ

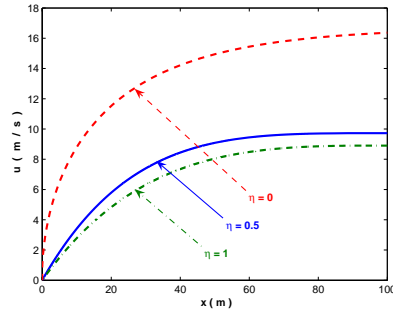


Fig. 8. Variation of velocity with x for different couple stress parameter η

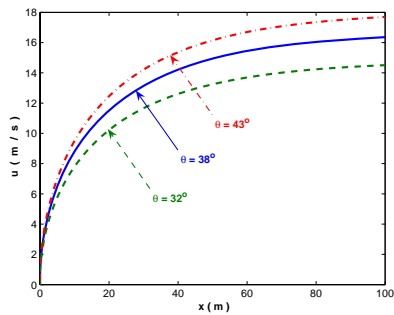


Fig. 5. Variation of velocity with x for different Slope angle θ

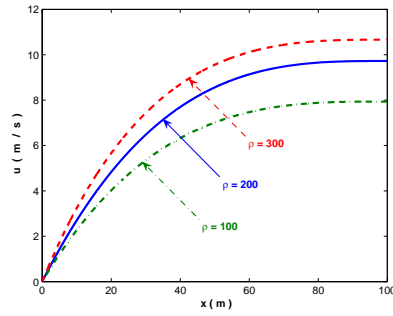


Fig. 9. Variation of velocity with x for different Snow density ρ (with $\eta = 0.5$)

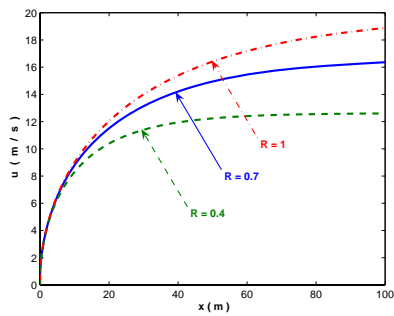


Fig. 6. Variation of velocity with x for different Slab depth R

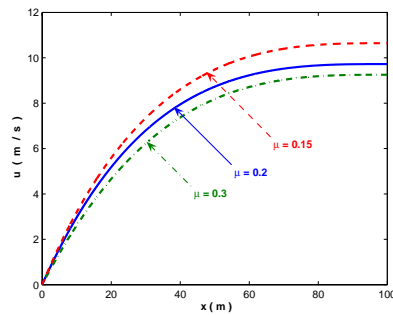


Fig. 10. Variation of velocity with x for different Viscosity μ (with $\eta = 0.5$)

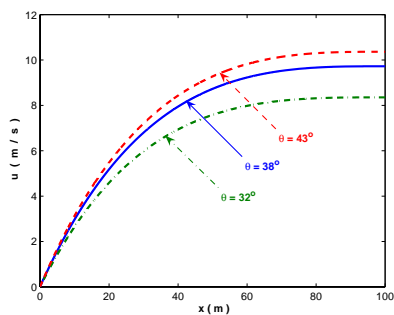


Fig. 11. Variation of velocity with x for different Slope angle θ (with $\eta = 0.5$)

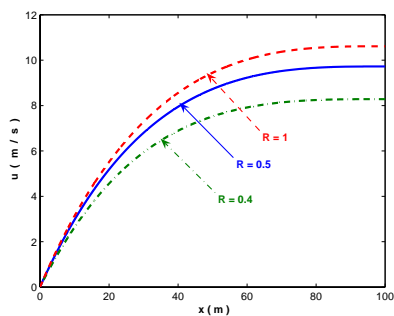


Fig. 12. Variation of velocity with x for different Slab depth R (with $\eta = 0.5$)

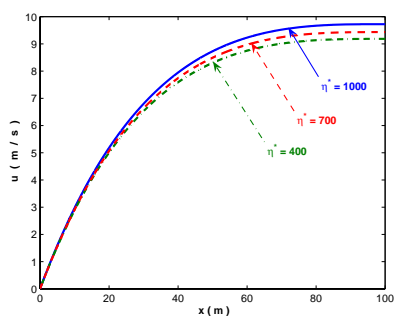


Fig. 13. Variation of velocity with x for different Eddy viscosity η^* (with $\eta = 0.5$)

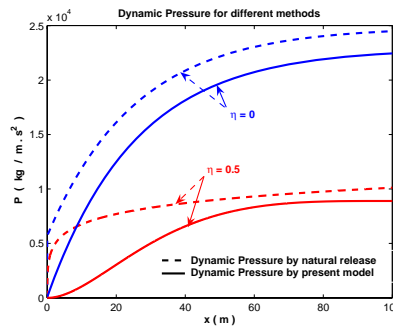


Fig. 14. Variation of dynamic pressure with distance (down the slope) for different couple stress parameters in Start zone to Track zone (compare present with natural release [14])

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avalanches. The results indicate that, the present computed results are lower in comparison to that of naturally released one and effects of couple stresses still lowers the magnitude of avalanche velocity. The results of the present finding are found to be in agreement with general physical observations. The present findings find its relevance in the design concept of control structures.

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