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Two-Stage Compensator Designs with Partial Feedbacks

Kazuyoshi MORI

Abstract—The two-stage compensator designs of linear system are investigated in the framework of the factorization approach. First, we give "full feedback" two-stage compensator design. Based on this result, various types of the two-stage compensator designs with partial feedbacks are derived.

Keywords—Linear System, Factorization Approach, Two-Stage Compensator Design, Parametrization of Stabilizing Controllers.

I. Introduction

The factorization approach to control systems has the advantage that it embraces, within a single framework, numerous linear systems such as continuous-time as well as discrete-time systems, lumped as well as distributed systems, one-dimensional as well as multidimensional systems, etc.[1], [2], [3]. Hence the result given in this paper will be able to a number of models in addition to the multidimensional systems. In factorization approach, when problems such as feedback stabilization are studied, one can focus on the key aspects of the problem under study rather than be distracted by the special features of a particular class of linear systems. This approach leads to conceptually simple and computationally tractable solutions to many important and interesting problems[4]. A transfer matrix of this approach is considered as the ratio of two stable causal transfer matrices. For a long time, the theory of the factorization approach had been founded on the coprime factorizability of transfer matrices, which is satisfied by transfer matrices over the principal ideal domains or the Bézout domains.

In some design problems, one uses a so-called *two-state procedure* for selecting an appropriate stabilizing compensator[4]. Given a plant, the first stage consists of selecting a stabilizing compensator for the plant. The second stage consists of selecting a stabilizing controller for the new closed-loop system that also achieves some other design objectives such as decoupling, sensitivity minimization, etc. The rationale behind this procedure is that the design problems are often easier to solve when the plant is stable. So far, the results of the two-stage compensator design use the norm algebras as well as the factorization approach. Because the analysis by the norm algebra is based on a concrete specified model, this reduces the attractiveness of the factorization approach.

First, we present a two-stage compensator design based on "full feedback" (Theorem 3). Using this result, we will present various types of the two-stage compensator designs as subsets of "full feedback" (Theorems 4 to 7).

School of Computer Science and Engineering, The University of Aizu, Aizu-Wakamatsu 965-8580, JAPAN.
Email: Kazuyoshi.MORI@IEEE.ORG

II. Preliminaries

The stabilization problem considered in this paper follows that of [5], and [6], who consider the feedback system Σ [4, Ch.5, Fig. 5.1] as in Fig. 1. For further details the reader is referred to [4], [7], [5], and [6].

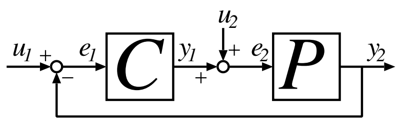

Fig. 1. Feedback system Σ.

We consider that the set of stable causal transfer functions is an integral domain, denoted by $\mathcal{A}$. The total ring of fractions of $\mathcal{A}$ is denoted by $\mathcal{F}$; that is, $\mathcal{F} = \{n/d \,|\, n, d \in \mathcal{A},\ d \neq 0\}$. This $\mathcal{F}$ is considered as the set of all possible transfer functions. Matrices over $\mathcal{F}$ are transfer matrices. Let $\mathcal{Z}$ be a prime ideal of $\mathcal{A}$ with $\mathcal{Z} \neq \mathcal{A}$. Define the subsets $\mathcal{P}$ and $\mathcal{P}_{\mathrm{S}}$ of $\mathcal{F}$ as follows: $\mathcal{P} = \{a/b \in \mathcal{F} \,|\, a \in \mathcal{A},\ b \in \mathcal{A}\backslash\mathcal{Z}\}$, $\mathcal{P}_{\mathrm{S}} = \{a/b \in \mathcal{F} \,|\, a \in \mathcal{Z},\ b \in \mathcal{A}\backslash\mathcal{Z}\}$. Then, every transfer function in $\mathcal{P}$ ($\mathcal{P}_{\mathrm{S}}$) is called *causal* (*strictly causal*). Analogously, if every entry of a transfer matrix is in $\mathcal{P}$ ($\mathcal{P}_{\mathrm{S}}$), the transfer matrix is called *causal* (*strictly causal*).

Throughout the paper, the plant we consider has m inputs and n outputs, and its transfer matrix, which is also called a *plant* itself simply, is denoted by P and belongs to $\mathcal{P}^{n\times m}$. We can always represent P in the form of a fraction $P = ND^{-1}$ ($P = \widetilde{D}^{-1}\widetilde{N}$), where $N \in \mathcal{A}^{n\times m}$ ($\widetilde{N} \in \mathcal{A}^{n\times m}$) and $D \in \mathcal{A}^{m\times m}$ ($\widetilde{D} \in \mathcal{A}^{n\times n}$) with nonsingular D ($\widetilde{D}$).

For $P \in \mathcal{F}^{n\times m}$ and $C \in \mathcal{F}^{m\times n}$, a matrix $H(P, C) \in \mathcal{F}^{(m+n)\times(m+n)}$ is defined as

$$H(P,C) := \begin{bmatrix} (I_n + PC)^{-1} & -P(I_m + CP)^{-1} \\ C(I_n + PC)^{-1} & (I_m + CP)^{-1} \end{bmatrix} \qquad (1)$$

provided that $\det(I_n + PC)$ is a nonzero of $\mathcal{A}$. This $H(P, C)$ is the transfer matrix from $[\,u_1^t \quad u_2^t\,]^t$ to $[\,e_1^t \quad e_2^t\,]^t$ of the feedback system Σ. If $\det(I_n + PC)$ is a nonzero of $\mathcal{A}$ and $H(P, C) \in \mathcal{A}^{(m+n)\times(m+n)}$, then we say that the plant P is *stabilizable*, P is *stabilized* by C, and C is a *stabilizing controller* of P. In the definition above, we do not mention the causality of the stabilizing controller. However, it is known that if a causal plant is stabilizable, there always exists a causal stabilizing controller of the plant [6].

It is known that $W(P, C)$ defined below is over $\mathcal{A}$ if and only if $H(P,C)$ is over $\mathcal{A}$:

$$W(P,C) := \qquad (2)$$
$$\begin{bmatrix} C(I_n+PC)^{-1} & -CP(I_m+CP)^{-1} \\ PC(I_n+PC)^{-1} & P(I_m+CP)^{-1} \end{bmatrix}.$$

This $W(P,C)$ is the transfer matrix from u_1 and u_2 to y_1 and y_2.

We employ the factorization approach [1], [8], [4], [2] and the symbols used in [9] and [5]. Also we will denote by $\mathcal{S}(P)$ the set of stabilizing controllers of P and by $\mathcal{W}(P)$ the set of all $W(P,C)$'s with $C \in \mathcal{S}(P)$.

III. Two-Stage Compensator Design

In some design problems, one uses a so-called *two-state procedure* for selecting an appropriate stabilizing compensator[4]. Given a plant P, the first stage consists of selecting a stabilizing compensator for P. Let $C_0 \in S(P)$ denote this compensator (that is, an arbitrary but fixed compensator of P) and define $P_1 = P(I+C_0P)^{-1}$. The second stage consists of selecting a stabilizing controller for P_1 that also achieves some other design objectives such as decoupling, sensitivity minimization, etc. The rationale behind this procedure is that the design problems are often easier to solve when the plant is stable. The resulting configuration with its inner and outer loops is shown in Figure 2.

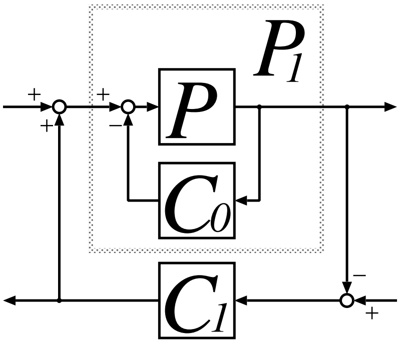

Fig. 2. Two-Stage Compensator Design (y_2 to u_2).

Here we give that the two-stage compensator design based on Figure 2 cannot give all stabilizing controllers. The following Theorem 1 is same as Theorem 5.3.10 of [4]. More detailed result can be found in Mori[10], which is shown as Theorem 2.

Theorem 1: Let P denote a causal plant of $\mathcal{P}^{n\times m}$ and C_0 a causal stabilizing controller of P ($C_0 \in \mathcal{P}^{m\times n}$). Further let P_1 be $P(I_m+C_0P)^{-1}$. Denote by $C_0+\mathcal{S}(P_1)$ the following set:

$$\{C_0+C_1 \,|\, C_1 \in \mathcal{S}(P_1)\}.$$

Then

$$C_0+\mathcal{S}(P_1) \subset \mathcal{S}(P), \qquad (3)$$

with equality holding if and only if $C_0 \in \mathcal{A}^{m\times n}$.

Theorem 2: Let P, C_0, P_1 be as in Theorem 1.

Let N, D, $\widetilde{N}$, $\widetilde{D}$, Y, X, $\widetilde{Y}$, $\widetilde{X}$ be matrices over $\mathcal{A}$ such that

$$\begin{cases} P = ND^{-1} = \widetilde{D}^{-1}\widetilde{N}, \quad C_0 = YX^{-1} = \widetilde{X}^{-1}\widetilde{Y}, \\ \widetilde{Y}N + \widetilde{X}D = I, \quad \widetilde{N}Y + \widetilde{D}X = I. \end{cases} \qquad (4)$$

Then we have

$$\begin{aligned} & C_0+\mathcal{S}(P_1) && (5)\\ &= \{(\widetilde{X}-R\widetilde{N})^{-1}(\widetilde{Y}+R\widetilde{D}) \,|\, R=\widetilde{X}R_1X, R_1 \in \mathcal{A}^{m\times n}\} && (6)\\ &= \{(Y+DR)(X-NR)^{-1} \,|\, R=\widetilde{X}R_1X, R_1 \in \mathcal{A}^{m\times n}\} && (7)\end{aligned}$$

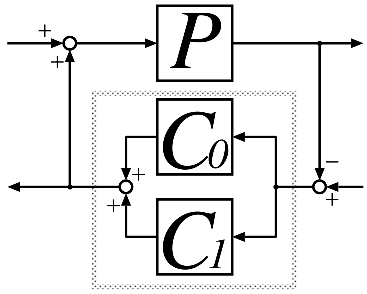

Fig. 3. Composite Stabilized Feedback with C_0 and C_1.

By Theorem 1, we see that the sum of C_0 and a stabilizing controller of P_1, say C_1, is again a stabilizing controller of P. This sum, a stabilizing controller of P, is the parallel allocation of C_0 and C_1, as shown in Figure 3.

The stabilizing controller C_0 is over $\mathcal{A}$ if and only if the matrices $\widetilde{X}$ and X are unimodular. That is the equality of (3) holds if and only if C_0 is over $\mathcal{A}$.

IV. "Full Feedback" Two-Stage Compensator Design

The two-stage compensator design of the previous section was based on the feedback from y_2 to u_2 (cf. Figures 1 and 2). Even so, we note that we have two inputs u_1 and u_2 and two outputs y_1 and y_2. Thus we consider the feedback as shown in Figure 4. This is a feedback from y_1 and y_2 to u_1 and u_2.

In Figure 4, C_0 is a stabilizing controller of P and C_1 is a stabilizing controller of $W(P,C_0)$. By noting that $W(P,C_0)$ is over $\mathcal{A}$, the parametrization of C_1 is given as

$$\begin{aligned} C_1 &= (I_{m+n} - R_1W(P,C_0))^{-1}R_1 \qquad (8)\\ &= R_1(I_{m+n} - W(P,C_0)R_1)^{-1} \end{aligned}$$

with a parameter $R_1 \in \mathcal{A}^{(m+n)\times(m+n)}$.

We next decompose this C_1 as follows:

$$C_1 = \begin{array}{c c} & \begin{array}{cc} m & n \end{array} \\ \begin{array}{c} n \\ m \end{array} & \begin{bmatrix} C_{111} & C_{112} \\ C_{121} & C_{122} \end{bmatrix} \end{array}.$$

where $C_{111} \in \mathcal{P}^{n\times m}$, $C_{112} \in \mathcal{P}^{n\times n}$, $C_{121} \in \mathcal{P}^{m\times m}$, and $C_{122} \in \mathcal{P}^{m\times n}$. Also we next decompose R_1 as follows:

$$R_1 = \begin{array}{c c} & \begin{array}{cc} m & n \end{array} \\ \begin{array}{c} n \\ m \end{array} & \begin{bmatrix} R_{111} & R_{112} \\ R_{121} & R_{122} \end{bmatrix} \end{array},$$

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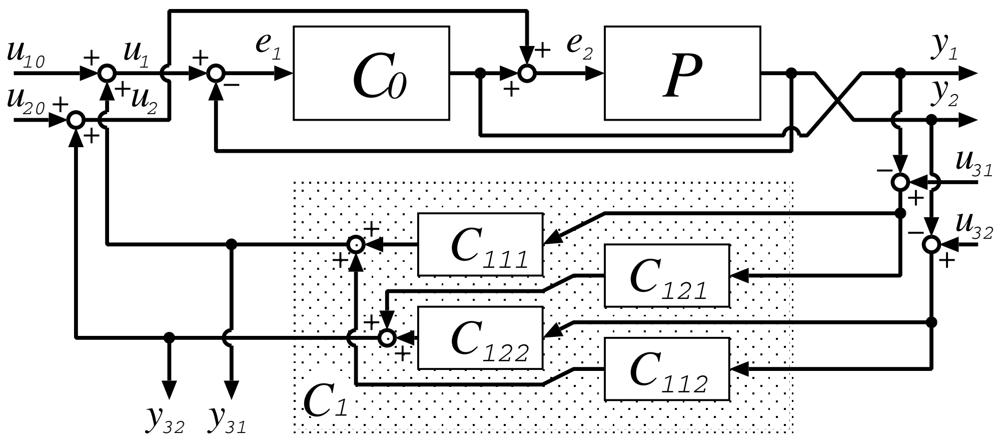

Fig. 4. Composite Stabilized Feedback with C_0 and C_1.

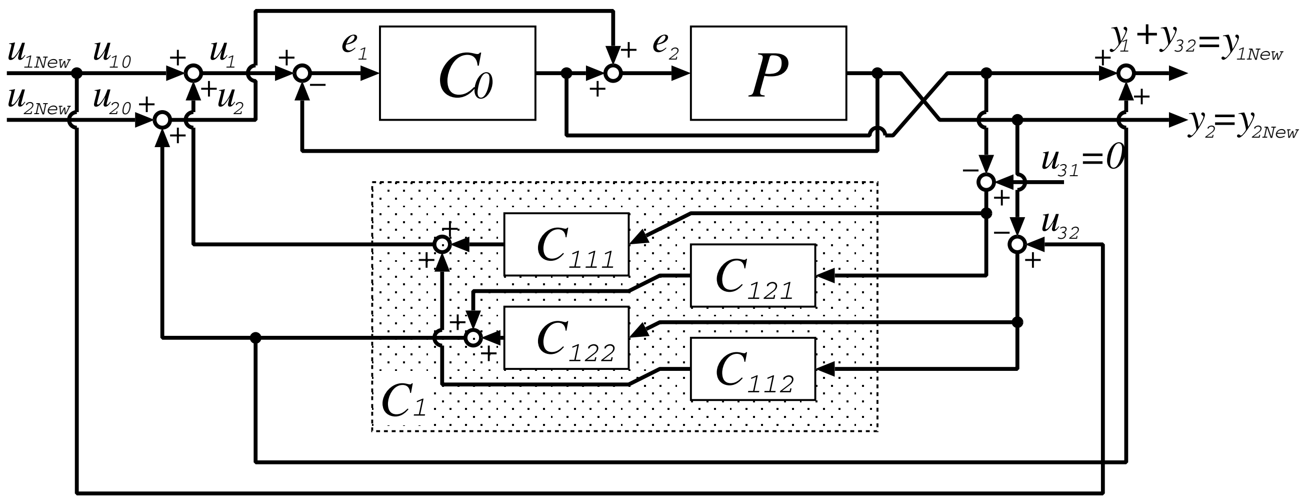

Fig. 5. Feedback System with C_0 and C_1.

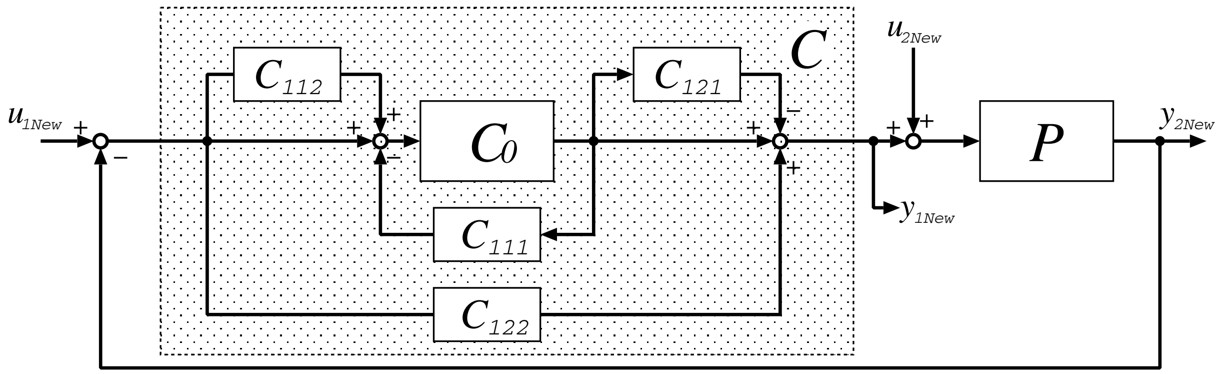

Fig. 6. Reconfigured Feedback System

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TABLE I
FEEDBACK FROM OUTPUTS TO INPUTS

No.	Input(s)	Output(s)
1	u_1	y_1
2	u_1	y_2
3	u_2	y_1
4	u_2	y_2
5	u_1, u_2	y_1
6	u_1, u_2	y_2
7	u_1	y_1, y_2
8	u_2	y_1, y_2
9	u_1, u_2	y_1, y_2

where $R_{111} \in \mathcal{A}^{n\times m}$, $R_{112} \in \mathcal{A}^{n\times n}$, $R_{121} \in \mathcal{A}^{m\times m}$, and $R_{122} \in \mathcal{A}^{m\times n}$. Using these parameters, we have the following theorem.

Theorem 3: Any stabilizing controller C of P is in the form $C = C_n C_d^{-1}$, where

$$C_n = \widetilde{Y} + (\widetilde{Y}R_{112}X + \widetilde{X}R_{122}X - \widetilde{Y}R_{111}Y - \widetilde{X}R_{121}Y)\widetilde{D}, \quad (9)$$

$$C_d = \widetilde{X} - (\widetilde{Y}R_{112}X + \widetilde{X}R_{122}X - \widetilde{Y}R_{111}Y - \widetilde{X}R_{121}Y)\widetilde{N}. \quad (10)$$

Proof: (Due to the space limitation, we give a brief proof only.) First we consider the feedback system as in Figure 5. This can be reconfigured as in Figure 6.

From the matrix computation with C in the description of Theorem 3, we have a matrix equation:

$$W(P,C) = \begin{bmatrix} O & I_m & I_m & O \\ O & O & O & I_n \end{bmatrix} W(W(P,C_0),C_1) \begin{bmatrix} O & O \\ I_n & O \\ I_n & O \\ O & I_m \end{bmatrix}.$$

Thus C is a stabilizing controller of the plant P.

On the other hand, by letting

$$\begin{aligned} R_{111} &= -NR\widetilde{N}, \\ R_{112} &= NR\widetilde{D}, \\ R_{121} &= -DR\widetilde{N}, \\ R_{122} &= DR\widetilde{D}, \end{aligned}$$

we have

$$C = (\widetilde{Y} + R\widetilde{D})(\widetilde{X} - R\widetilde{N})^{-1},$$

This means any stabilizing controller can be expressed as in (9) and (10). ∎

V. PARTIAL TWO-STAGE COMPENSATOR DESIGNS

From Theorem 3, we can derive two-stage compensator designs based on various feedback styles. From Figure 1, we have two inputs u_1 and u_2, and two outputs y_1 and y_2. Thus we have 9 types of feedbacks, which are shown in Table I. For each type of feedback, we can obtain a two-stage compensator design.

In the following, we first, review the two-stage compensator design based on the feedback from y_2 to u_2 (No. 4). Then we give the two-stage compensator designs based on the feedbacks of Nos. 5 to 8. The feedback of No. 9 is just Theorem 3.

(Review) Feedback from y_2 to u_2 (No. 4)

Let us review the two-stage compensator design of Section III, which is based on the feedback from y_2 to u_2. In this case, the feedback system is as in Figure 2. Under the current situation, we have used new symbol C_{122} instead of C_1, which is shown in Figure 7.

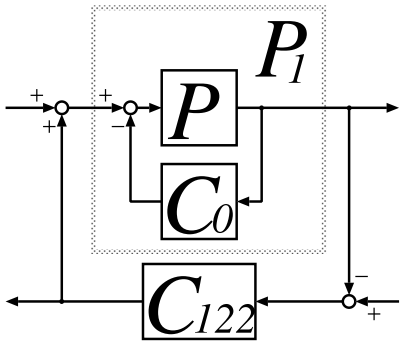

Fig. 7. Feedback from y_2 to u_2.

This is the case where C_{111}, C_{112}, and C_{121} of Figures 5 and 6 are zero matrices. From $(I_{m+n} - R_1W(P,C_0))^{-1}R_1$ of (8), R_{111} and R_{121} are zero matrices. Analogously, from $R_1(I_{m+n} - W(P,C_0)R_1)^{-1}$ of (8), R_{111} and R_{112} are zero matrices. Thus only R_{122} can be nonzero matrix and other R_{111}, R_{112}, and R_{121} are zero matrices. This implies Theorem 2.

The procedure to obtain a stabilizing controller based on this feedback (No. 4) is summarized below.

Input
P: A plant to be stabilized ($\in \mathcal{P}^{n\times m}$).
C_0: A stabilizing controller ($\in \mathcal{F}^{m\times n}$).
R_1: A parameter matrix ($\in \mathcal{A}^{m\times n}$).
Output
C_{New}: A new stabilizing controller ($\in \mathcal{F}^{m\times n}$).
Procedure
(i) Let $P_1 = P(I_m + C_0P)^{-1}$.
(ii) Assume $I_m - R_1P_1$ is nonsingular.
(iii) Let $C_1 = (I_m - R_1P_1)^{-1}R_1$.
(iv) Let $C_{\mathrm{New}} = C_0 + C_1$.
(v) Return (C_{New}).

Feedback from y_1 to u_1 and u_2 (No. 5)

Let us consider the two-stage compensator design based on the feedback from y_1 to u_1 and u_2. That is, we do not consider the output u_2. In this case, the feedback system is as in Figure 8.

This is the case where C_{112} and C_{122} of Figures 5 and 6 are zero matrices. From $(I_{m+n} - R_1W(P,C_0))^{-1}R_1$ of (8), R_{112} and R_{122} are zero matrices. Thus only R_{111} and R_{121} can be nonzero matrix. Hence the configuration is as in Figure 9.

The following is a derivative result of Theorem 3 based on the feedback from y_1 to u_1 and u_2.

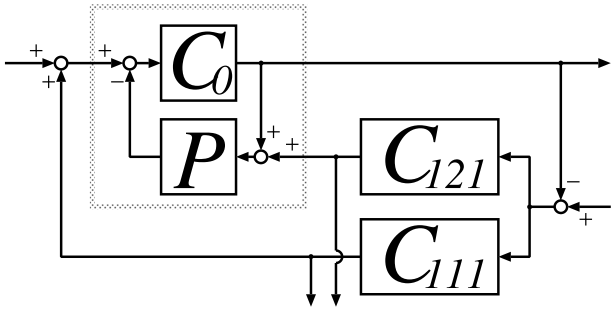

Fig. 8. Feedback from y_1 to u_1 and u_2.

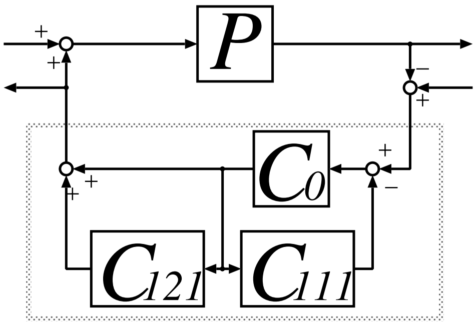

Fig. 9. Composite Stabilized Feedback based on Feedback from y_1 to u_1 and u_2.

Theorem 4: Let $P_{y_1u_1u_2}$ denote

$$[\,C_0(I_n+PC_0)^{-1} \quad -C_0P(I_m+C_0P)^{-1}\,].$$

Then we have

$$\begin{aligned} &\{(I_n+C_{121})(I_m+C_0C_{111})^{-1}C_0 \mid \\ &\qquad \begin{bmatrix} C_{111} \\ C_{121} \end{bmatrix} \in \mathcal{S}(P_{y_1u_1u_2}), \\ &\qquad |I_m+C_0C_{111}| \neq 0\} \\ &= \{(\widetilde{X}-R\widetilde{N})^{-1}(\widetilde{Y}+R\widetilde{D}) \mid \\ &\qquad R=-\widetilde{Y}R_{111}Y-\widetilde{X}R_{121}Y, \\ &\qquad R_{111}\in\mathcal{A}^{n\times m}, R_{121}\in\mathcal{A}^{m\times m}, \\ &\qquad |\widetilde{X}-R\widetilde{N}|\neq 0\} \\ &= \{(\widetilde{X}-R\widetilde{N})^{-1}(\widetilde{Y}+R\widetilde{D}) \mid \\ &\qquad R=R'Y, R'\in\mathcal{A}^{m\times m}, \\ &\qquad |\widetilde{X}-R\widetilde{N}|\neq 0\}. \end{aligned} \quad (11)$$

The procedure to obtain a stabilizing controller based on this feedback is summarized below.

Input
P: A plant to be stabilized ($\in \mathcal{P}^{n\times m}$).
C_0: A stabilizing controller ($\in \mathcal{F}^{m\times n}$).
R_1: A parameter matrix ($\in \mathcal{A}^{(m+n)\times m}$).
Output
$C_{\rm New}$: A new stabilizing controller ($\in \mathcal{F}^{m\times n}$).
Procedure
(i) Let $P_1 = [\,C_0(I_n+PC_0)^{-1} \quad -C_0P(I_m+C_0P)^{-1}\,]$.
(ii) Let $C_1 = (I_{m+n}-R_1P_1)^{-1}R_1$.
(iii) Decompose $[\,C_{111}^t \quad C_{121}^t\,]^t := C_1$
with $C_{111}\in\mathcal{F}^{n\times m}$ and $C_{121}\in\mathcal{F}^{m\times m}$.
(iv) Assume $I_m+C_0C_{111}$ is nonsingular.
(v) Let $C_{\rm New} = (I_m+C_{121})(I_m+C_0C_{111})^{-1}C_0$.
(vi) Return ($C_{\rm New}$).

Feedback from y_2 to u_1 and u_2 (No. 6)

The feedback of No. 5 described above has employed the output y_1. Alternatively we now consider to use the output y_2. That is, consider the two-stage compensator design based on the feedback from y_2 to u_1 and u_2. In this case, the feedback system is as in Figure 10.

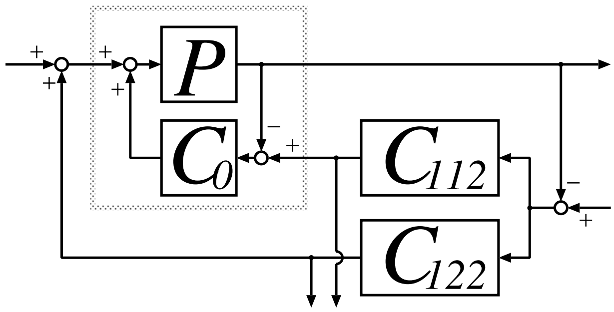

Fig. 10. Feedback from y_2 to u_1 and u_2.

This is the case where C_{111} and C_{121} of Figures 5 and 6 are zero matrices. From $(I_{m+n}-R_1W(P,C_0))^{-1}R_1$ of (8), R_{111} and R_{121} are zero matrices. Thus only R_{112} and R_{122} can be nonzero matrix. Hence the configuration is as in Figure 11.

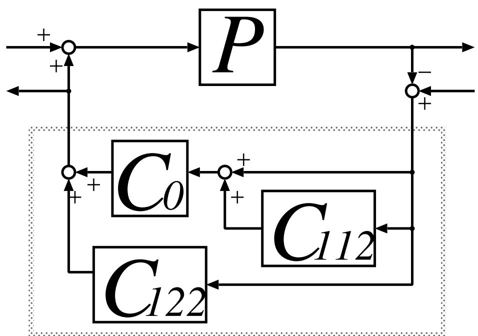

Fig. 11. Composite Stabilized Feedback based on Feedback from y_2 to u_1 and u_2.

The following is a derivative result of Theorem 3 based on the feedback from y_1 to u_1 and u_2.

Theorem 5: Let $P_{y_2u_1u_2}$ denote

$$[\,PC_0(I_n+PC_0)^{-1} \quad P(I_m+C_0P)^{-1}\,].$$

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Then we have

$$\begin{aligned}
&\{C_0(I_n+C_{112})+C_{122} \mid \begin{bmatrix} C_{112} \\ C_{122} \end{bmatrix} \in \mathcal{S}(P_{y_2u_1u_2})\} \\
&= \{(\widetilde{X}-R\widetilde{N})^{-1}(\widetilde{Y}+R\widetilde{D}) \mid \\
&\qquad R=\widetilde{Y}R_{112}X+\widetilde{X}R_{122}X, \\
&\qquad R_{112}\in\mathcal{A}^{n\times n}, R_{122}\in\mathcal{A}^{m\times n}, \\
&\qquad |\widetilde{X}-R\widetilde{N}|\neq 0\} \\
&= \{(\widetilde{X}-R\widetilde{N})^{-1}(\widetilde{Y}+R\widetilde{D}) \mid \\
&\qquad R=R'X, R'\in\mathcal{A}^{m\times n}, \\
&\qquad |\widetilde{X}-R\widetilde{N}|\neq 0\}.
\end{aligned} \tag{12}$$

The procedure to obtain a stabilizing controller based on this feedback is summarized below.

Input
P: A plant to be stabilized ($\in \mathcal{P}^{n\times m}$).
C_0: A stabilizing controller ($\in \mathcal{F}^{m\times n}$).
R_1: A parameter matrix ($\in \mathcal{A}^{(m+n)\times n}$).
Output
C_{New}: A new stabilizing controller ($\in \mathcal{F}^{m\times n}$).
Procedure
(i) Let $P_1 = [\, PC_0(I_n+PC_0)^{-1} \quad P(I_m+C_0P)^{-1}\,]$.
(ii) Let $C_1 = (I_{m+n}-R_1P_1)^{-1}R_1$.
(iii) Decompose $[\, C_{112}^t \quad C_{122}^t \,]^t := C_1$
with $C_{112}\in\mathcal{F}^{n\times n}$ and $C_{122}\in\mathcal{F}^{m\times n}$.
(iv) Let $C_{\mathrm{New}} = C_0(I_n+C_{112})+C_{122}$.
(v) Return (C_{New}).

Feedback from y_1 and y_2 to u_1 (No. 7)

The feedbacks of Nos. 5 and 6 described above has employed two inputs. In the following, we consider one inputs and two outputs. First, we consider the feedback from y_1 and y_2 to u_1.

In this case, the feedback system is as in Figure 12.

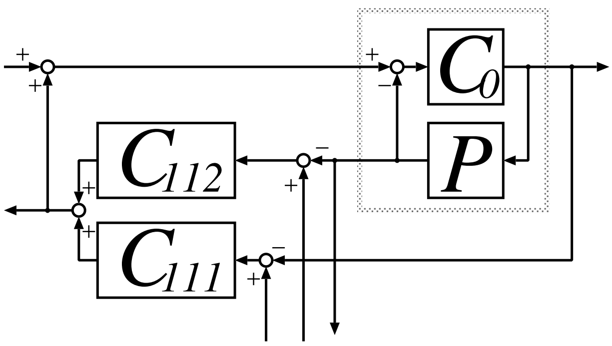

Fig. 12. Feedback from y_1 and y_2 to u_1.

This is the case where C_{121} and C_{122} of Figures 5 and 6 are zero matrices. From $R_1(I_{m+n}-W(P,C_0)R_1)^{-1}$ of (8), R_{121} and R_{122} are zero matrices. Thus only R_{111} and R_{112} can be nonzero matrix. Hence the configuration is as in Figure 13.

The following is a derivative result of Theorem 3 based on the feedback from y_1 and y_1 to u_1.

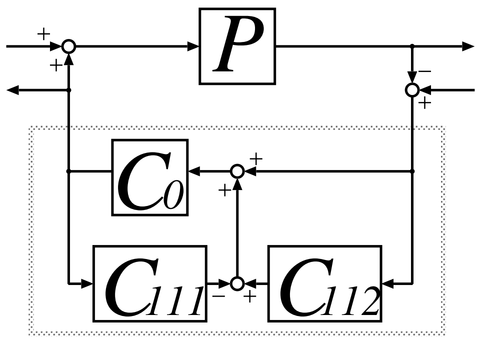

Fig. 13. Composite Stabilized Feedback based on Feedback from y_1 and y_2 to u_1.

Theorem 6: Let $P_{y_1y_2u_1}$ denote

$$\begin{bmatrix} C_0(I_n+PC_0)^{-1} \\ PC_0(I_n+PC_0)^{-1} \end{bmatrix}.$$

Then we have

$$\begin{aligned}
&\{(I_m+C_0C_{111})^{-1}C_0(I_n+C_{112}) \mid \\
&\qquad [\, C_{111} \quad C_{112}\,] \in \mathcal{S}(P_{y_1y_2u_1})\} \\
&= \{(\widetilde{X}-R\widetilde{N})^{-1}(\widetilde{Y}+R\widetilde{D}) \mid \\
&\qquad R=\widetilde{Y}R_{112}X-\widetilde{Y}R_{111}Y, \\
&\qquad R_{112}\in\mathcal{A}^{n\times n}, R_{111}\in\mathcal{A}^{n\times m}, \\
&\qquad |\widetilde{X}-R\widetilde{N}|\neq 0\} \\
&= \{(\widetilde{X}-R\widetilde{N})^{-1}(\widetilde{Y}+R\widetilde{D}) \mid \\
&\qquad R=\widetilde{Y}R', R'\in\mathcal{A}^{n\times n}, \\
&\qquad |\widetilde{X}-R\widetilde{N}|\neq 0\}.
\end{aligned} \tag{13}$$

The procedure to obtain a stabilizing controller based on this feedback is summarized below.

Input
P: A plant to be stabilized ($\in \mathcal{P}^{n\times m}$).
C_0: A stabilizing controller ($\in \mathcal{F}^{m\times n}$).
R_1: A parameter matrix ($\in \mathcal{A}^{n\times(m+n)}$).
Output
C_{New}: A new stabilizing controller ($\in \mathcal{F}^{m\times n}$).
Procedure
(i) Let $P_1 = \begin{bmatrix} C_0(I_n+PC_0)^{-1} \\ PC_0(I_n+PC_0)^{-1} \end{bmatrix}$.
(ii) Let $C_1 = R_1(I_{m+n}-P_1R_1)^{-1}$.
(iii) Decompose $[\, C_{111} \quad C_{112}\,] := C_1$
with $C_{111}\in\mathcal{F}^{n\times m}$ and $C_{112}\in\mathcal{F}^{n\times n}$.
(iv) Assume $I_m+C_0C_{111}$ is nonsingular.
(v) Let $C_{\mathrm{New}} = (I_m+C_0C_{111})^{-1}C_0(I_n+C_{112})$.
(vi) Return (C_{New}).

Feedback from y_1 and y_2 to u_2 (No. 8)

The final feedback is that from from y_1 and y_2 to u_2.

In this case, the feedback system is as in Figure 14.

This is the case where C_{111} and C_{112} of Figures 5 and 6 are zero matrices. From $R_1(I_{m+n}-W(P,C_0)R_1)^{-1}$ of (8), R_{111}

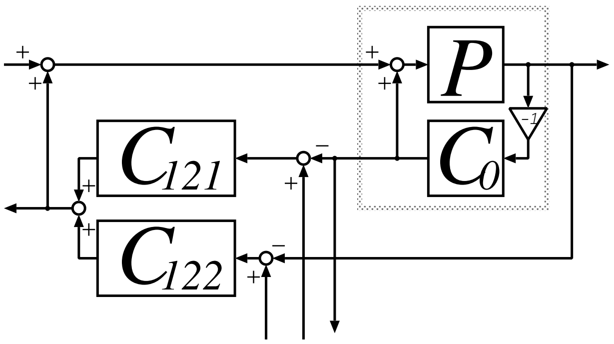

Fig. 14. Feedback from y_1 and y_2 to u_2.

and R_{112} are zero matrices. Thus only R_{121} and R_{122} can be nonzero matrix. Hence the configuration is as in Figure 15.

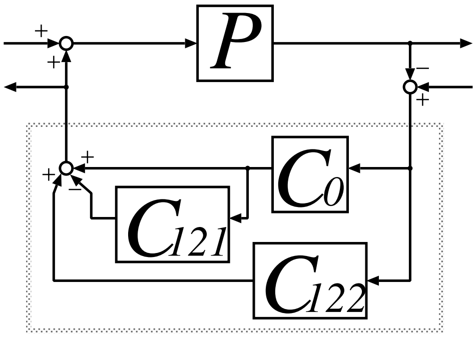

Fig. 15. Composite Stabilized Feedback based on Feedback from y_1 and y_2 to u_2.

The following is a derivative result of Theorem 3 based on the feedback from y_1 and y_1 to u_1.

Theorem 7: Let $P_{y_1 y_2 u_2}$ denote

$$\begin{bmatrix} -C_0P(I_m + C_0P)^{-1} \\ P(I_m + C_0P)^{-1} \end{bmatrix}.$$

Then we have

$$\begin{aligned} &\{(I_m + C_{121})C_0 + C_{122} \mid \\ &\qquad [\, C_{121} \quad C_{122} \,] \in \mathcal{S}(P_{y_1y_2u_2})\} \\ &= \{(\widetilde{X} - R\widetilde{N})^{-1}(\widetilde{Y} + R\widetilde{D}) \mid \\ &\qquad R = \widetilde{X}R_{122}X - \widetilde{X}R_{121}Y, \\ &\qquad R_{121} \in \mathcal{A}^{m\times m}, R_{122} \in \mathcal{A}^{m\times n}, \\ &\qquad |\widetilde{X} - R\widetilde{N}| \neq 0\} \\ &= \{(\widetilde{X} - R\widetilde{N})^{-1}(\widetilde{Y} + R\widetilde{D}) \mid \\ &\qquad R = \widetilde{X}R', R' \in \mathcal{A}^{m\times n}, \\ &\qquad |\widetilde{X} - R\widetilde{N}| \neq 0\}. \end{aligned} \tag{14}$$

The procedure to obtain a stabilizing controller based on this feedback is summarized below.

Input

P: A plant to be stabilized ($\in \mathcal{P}^{n\times m}$).
C_0: A stabilizing controller ($\in \mathcal{F}^{m\times n}$).
R_1: A parameter matrix ($\in \mathcal{A}^{m\times(m+n)}$).

Output

C_{New}: A new stabilizing controller ($\in \mathcal{F}^{m\times n}$).

Procedure

(i) Let $P_1 = \begin{bmatrix} -C_0P(I_m + C_0P)^{-1} \\ P(I_m + C_0P)^{-1} \end{bmatrix}$.
(ii) Let $C_1 = R_1(I_{m+n} - P_1R_1)^{-1}$.
(iii) Decompose $[\, C_{121} \quad C_{122} \,] := C_1$
with $C_{121} \in \mathcal{F}^{m\times m}$ and $C_{122} \in \mathcal{F}^{m\times n}$.
(iv) Let $C_{\text{New}} = (I_m + C_{121})C_0 + C_{122}$.
(v) Return (C_{New}).

VI. Conclusion

In this paper, we have investigated two-stage compensator designs. We have given five two-stage compensator designs with partial feedbacks. All results are given based on the factorization approach, so that the results can be applied to numerous linear systems.

As future works, we will consider to obtain the optimized stabilizing controller within the framework of two-stage compensator designs. Our result of this paper have shown that the two-stage compensator designs with partial feedbacks cannot give, in general, all stabilizing controllers. Even so, the optimized stabilizing controller may be included in the set prameterizd as in Theorems 4 to 7. We will need to investigate these criteria.

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