International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:8, No:9, 2014

Flow and Heat Transfer of a Nanofluid over a Shrinking Sheet

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***Abstract*—**The problem of laminar fluid flow which results from the shrinking of a permeable surface in a nanofluid has been investigated numerically. The model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis. A similarity solution is presented which depends on the mass suction parameter *S*, Prandtl number *Pr*, Lewis number *Le*, Brownian motion number *Nb* and thermophoresis number *Nt*. It was found that the reduced Nusselt number is decreasing function of each dimensionless number.

***Keywords*—**Boundary layer, Nanofluid, Shrinking sheet, Brownian motion, Thermophoresis, Similarity solution.

I. Introduction

THE boundary layer flow of an incompressible fluid over a shrinking sheet has received considerable attention of modern day researchers because of its increasing application to many engineering system. One of the common applications of shrinking sheet problems is shrinking film. In packing of bulk products, shrinking film is very useful as it can be unwrapped easily with adequate heat [1]. Wang [2] first pointed out the flow over a shrinking sheet when he was working on the flow of a liquid film over an unsteady stretching sheet. Then Miklavcic and Wang [3] found that the flow depends in externally imposed mass suction. The problem of stagnation point flow towards a shrinking sheet was studied by Wang [4] and found that the solutions do not exist for larger shrinking rates and non-unique in two-dimensional case. The unsteady viscous flow over a continuously shrinking surface with mass suction was also investigated by Fang et al. [5]. After that, Bhattacharyya [6] investigated the flow over exponentially shrinking sheet and found that the thermal boundary layer thickness becomes thinner due to the increasing Prandtl number.

A decade ago, with the rapid development of modern technology, particle of nanometre-size (normally less than 100 nm) are used for dispersing in base liquids, and they are called nanofluids that was first proposed by Choi [7]. Nanofluids containing nanoparticles and it have been shown to have higher heat transfer rates and thermal conductivity, even at very low solid concentrations. They are also very stable and have no additional problems such as erosion, additional pressure drop and non-Newtonion behavior, due to the tiny size of nanoelements and the low volume fraction of nanoelements required for conductivity enhancement. The enhanced thermal behavior of nanofluids could provide a basis for an enormous innovation for heat transfer intensification, which is major importance to a number of industrial sectors including transportation, nuclear reactors, electronics as well as biomedicine and food. There are some nanofluid models available in the literature. Among the popular models are the model proposed by Tiwari and Das [8] and Buongiorno [9] and. It is worth mentioning that the mathematical model proposed by Tiwari and Das [8] that was very recently used by Arifin et al. [10], Bachok et al. [11]-[13], Rohni et al. [14], Kameswaran et al. [15], and Das [16] in their papers. In the present paper, we study the boundary layer flow of a nanofluid and heat transfer over a shrinking sheet using the Buongiorno [9] model that analyzes the effects of Brownian motion and thermophoresis, which was also used by several authors, Nield and Kuznetsov [17], [18], Kuznetsov and Nield [19], [20], Khan and Pop [21], Bachok et al. [22], [23], Khan and Aziz [24], Nadeem and Lee [25].

The purpose of the present investigation is to study the boundary layer flow and heat transfer past a shrinking sheet in nanofluid. The governing partial differential equations are transformed into a set of ordinary differential equations using a similarity transformation, before being solved numerically by a shooting method. The results obtained are presented graphically and discussed.

II. Mathematical Formulation

Consider the flow of an incompressible nanofluid over a linearly shrinking sheet with suction at the boundary. The stretching/shrinking velocity $U_w(x) = ax$, where a is a constant and is maintained at a constant temperature T_w. It is also assumed that the mass flux velocity is V_0 with $V_0 > 0$ for injection and $V_0 < 0$ for suction. The simplified two-dimensional equations governing the flow in the boundary layer of a steady, laminar, and incompressible nanofluid are (see Khan and Pop [21])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

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$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_f}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \quad (2)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho_f}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \quad (3)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right] \quad (4)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2} \quad (5)$$

subject to the boundary conditions

$$u=\varepsilon U_w(x),\ v=V_0,\ T=T_w,\ C=C_w \quad \text{at } y=0,$$
$$u\to 0,\ T\to T_\infty,\ C\to C_\infty \quad \text{as } y\to\infty, \quad (6)$$

where u and v are the velocity components along x and y axes, respectively, $\alpha = k/(\rho c)_f$ is the thermal diffusivity of the fluid, υ is the kinematic coefficient, D_B is the Brownian diffusion coefficient, D_T is the thermophoresis diffusion coefficient and $\tau=(\rho c)_p/(\rho c)_f$ is the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid with ρ being the density, c is the volumetric volume expansion coefficient and ρ_p is the density of the particles. The stretching/shrinking parameter is ε with $\varepsilon=1$ for stretching and $\varepsilon=-1$ for shrinking.

The governing equations (1)–(5) subject to the boundary conditions (6) can be expressed in a simpler form by introducing the following transformation:

$$\eta=\left(\frac{a}{\nu}\right)^{1/2} y, \quad \psi=(\nu a)^{1/2}\, x\, f(\eta),$$
$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \quad (7)$$

where η is the similarity variable and ψ is the stream function defined as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$, which identically satisfies (1). Employing the similarity variables (7), (2), (3), (4) and (5) reduce to the following ordinary differential equations:

$$f'''+ff''-f'^2=0 \quad (8)$$

$$\frac{1}{Pr}\theta''+f\theta'+Nb\phi'\theta'+Nt\theta'^2=0 \quad (9)$$

$$\phi''+Le\, f\phi'+\frac{Nt}{Nb}\theta''=0 \quad (10)$$

subjected to the boundary conditions (5) which become

$$f(0)=S,\quad f'(0)=\varepsilon,\quad \theta(0)=1,\quad \phi(0)=1$$
$$f'(\eta)\to 0,\quad \theta(\eta)\to 0,\ \phi(\eta)\to 0 \quad \text{as } \eta\to\infty \quad (11)$$

where $S=-V_0/(\nu a)^{1/2}$ is the constant mass transfer parameter with $S>0$ for suction and $S<0$ for injection. In the above equations, primes denote differentiation with respect to η and the five parameters are defined by

$$\Pr=\frac{\nu}{\alpha},\quad Nb=\frac{(\rho C)_p D_B (C_w-C_\infty)}{(\rho C)_f\,\nu}$$
$$Le=\frac{\nu}{D_B},\quad Nt=\frac{(\rho C)_p D_T (T_w-T_\infty)}{(\rho C)_f T_\infty \nu} \quad (12)$$

where Pr is the Prandtl number, Le is the Lewis number, Nb is the Brownian motion parameter and Nt is the thermophoresis parameter.

The physical quantities of interest are the skin friction coefficient C_f, the local Nusselt number Nu_x and the local Sherwood number Sh_x which are defined as

$$C_f=\frac{\tau_w}{\rho U_w^2},\quad Nu_x=\frac{xq_w}{k(T_w-T_\infty)},\quad Sh_x=\frac{xq_m}{D_B(C_w-C_\infty)} \quad (13)$$

where the wall shear stress τ_w, the local heat flux q_w and the local mass flux q_m are given by

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0},\ q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0},\ q_m=-D_B\left(\frac{\partial C}{\partial y}\right)_{y=0} \quad (14)$$

with μ, k and D_B being the dynamic viscosity, thermal conductivity and the Brownian duffusion coefficient of the nanofluids, respectively. Using the similarity variables (7), we obtain

$$Re_x^{1/2}C_f = f''(0), \quad (15)$$

$$Re_x^{-1/2}Nu_x = -\theta'(0), \quad (16)$$

$$Re_x^{-1/2}Sh_x = -\phi'(0), \quad (17)$$

where $Re_x = U_w x/\nu$ is the local Reynolds number.

III. Results and Discussion

Numerical solutions to the governing ordinary differential equations (8)-(10) with the boundary conditions (11) were obtained using a shooting method. The analysis reveals the conditions for the existence of the steady boundary layer flow due to shrinking of the sheet and it is found that when the mass suction parameter S exceeds a certain critical value, say S_c, steady flow is possible. The similarity solution exists when the mass suction parameter *S* satisfies the condition $S \geq S_c$ and consequently for $S < S_c$ the flow has no similarity solution. Based on our computation, the critical value of *S* is 1.9999.

Different from the stretching case, which shows the unique solution, the solution for the shrinking case as presented in Figs. 1-4 are non-unique. The existence of dual solutions for the shrinking case was first reported by Miklavcic and Wang [3]. Figs. 1 and 2 shows the effects on *Le* and *Pr* numbers on the temperature distribution for selected values of *Nb*, *Nt* and *S* parameters. As expected, the boundary layer profiles for the temperature function $\theta(\eta)$ are essentially the same form as in the case of a regular fluid. It is observed that the temperature increases with the increase in both *Le* and *Pr* numbers. The effects of *Le* and *Pr* numbers on the concentration profiles for selected values of *Nb*, *Nt* and *S* parameters are shown in Figs. 3 and 4. It is clear that the concentration decreases as the Le and Pr numbers increase.

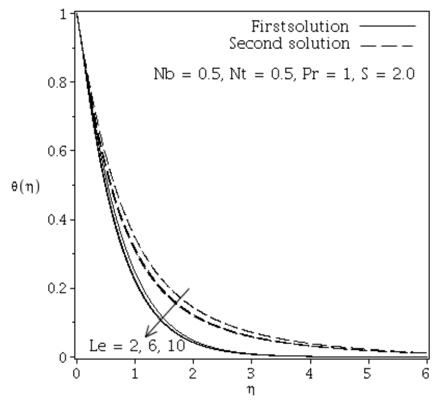


Fig. 1 Effect of *Le* number on temperature distribution for selected values of *Nb*, *Nt*, *Pr* and *S* when $\varepsilon = -1$ (shrinking case)

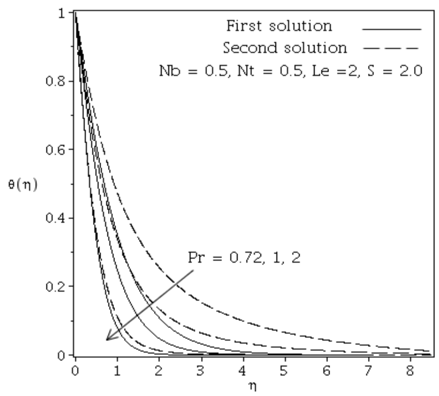


Fig. 2 Effect of *Pr* number on temperature distribution for selected values of *Nb*, *Nt*, *Le* and *S* when $\varepsilon = -1$ (shrinking case)

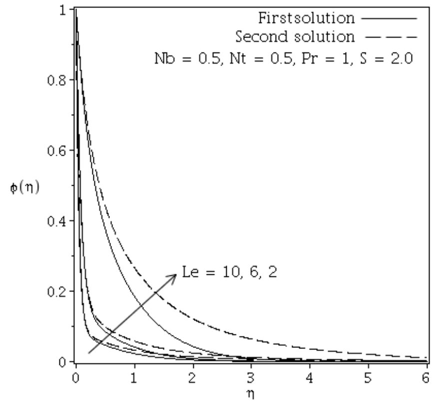


Fig. 3 Effect of *Le* number on concentration distribution for selected values of *Nb*, *Nt*, *Pr* and *S* when $\varepsilon = -1$ (shrinking case)

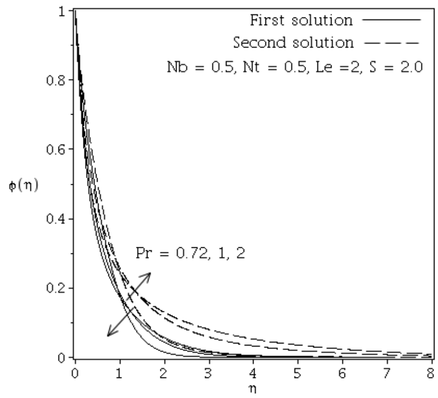
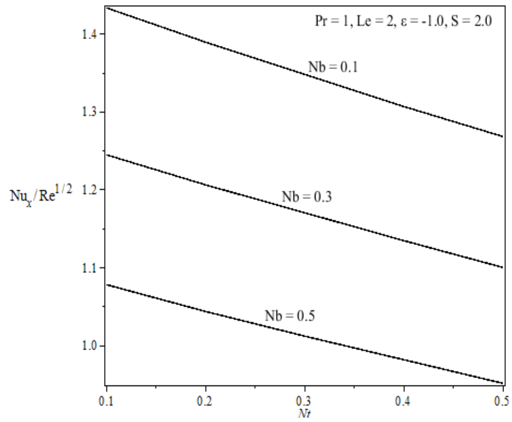
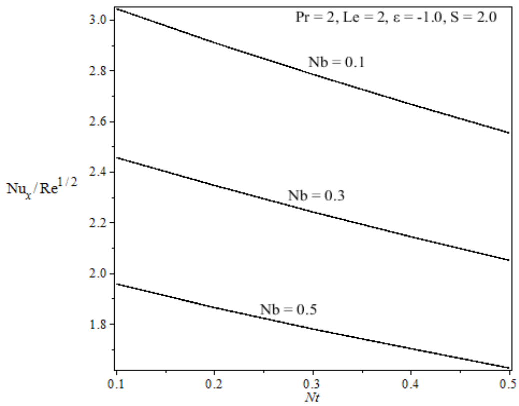


Fig. 4 Effect of *Pr* number on concentration distribution for selected values of *Nb*, *Nt*, *Le* and *S* when $\varepsilon = -1$ (shrinking case)

Next, the variation in dimensionless heat transfer rate *vs* *Nt* parameter is shown in Figs. 5 (a) and (b). They show that the effects of *Nb* parameters and *Pr* numbers on the dimensionless heat transfer rates for the same *Le* number. It is clear that the dimensionless heat transfer rate decrease with increase in *Nb* and *Nt* parameters but increase with the increase in *Pr* numbers. However, a decrease in the dimensionless heat transfer rates was observed with the increase in *Le* numbers. This is shown in Fig. 6. The change in the dimensionless heat transfer rates is found to be higher for smaller values of the parameter *Nb* and this change decreases with the increase of *Nt*.

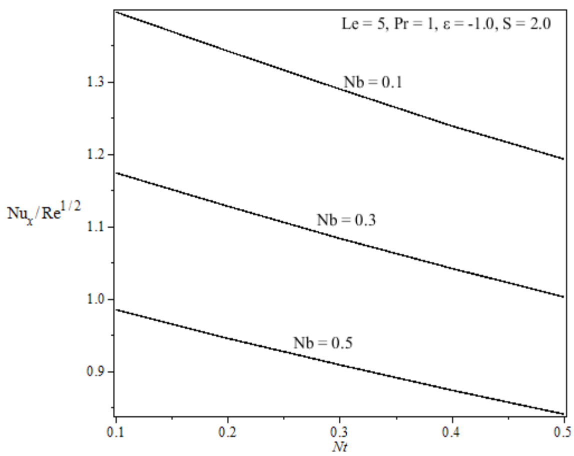


(a)

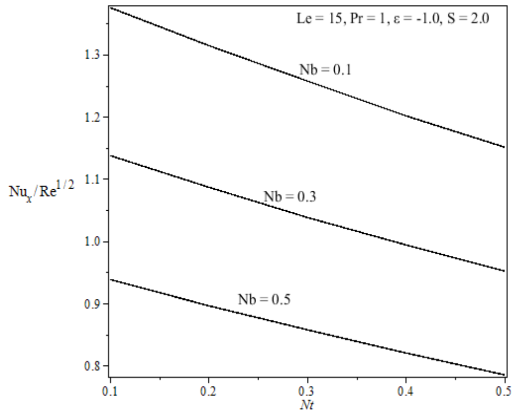


(b)

Fig. 5 Effect of *Nb* and *Pr* numbers on dimensionless heat transfer rates



(a)



(b)

Fig. 6 Effect of *Nb* and *Le* numbers on dimensionless heat transfer rates

IV. Conclusion

The characteristics of boundary layer flow and heat transfer past a permeable shrinking sheet in a nanofluid was investigated. The similarity equations were obtained and solved numerically by the shooting method. The study revealed that the steady boundary layer flow due to shrinking of the sheet is possible only when the mass suction parameter exceeds a certain value. It was found that the reduced Nusselt number is decreasing function of each value of the parameters *Pr*, *Le*, *Nb* and *Nt* considered.

Acknowledgment

The financial supports received from the Ministry of Higher Education, Malaysia (Project code: FRGS/1/2012/SG04/UPM/03/1) and the Research University Grant (RUGS) from the Universiti Putra Malaysia are gratefully acknowledged.

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