Using Interval Constrained Petri Nets and Fuzzy method for regulation of quality: the Case of Weight in Tobacco Factory

Nabli L., Dhouibi H., Collart Dutilleul S., Craye E.

Abstract—The existence of maximal durations drastically modifies the performance evaluation in Discrete Event Systems (DES). The same particularity may be found on systems where the associated constraints do not concern the time. For example weight measures, in chemical industry, are used in order to control the quantity of consumed raw materials. This parameter also takes a fundamental part in the product quality as the correct transformation process is based upon a given percentage of each essence. Weight regulation therefore increases the global productivity of the system by decreasing the quantity of rejected products. In this paper we present an approach based on mixing different characteristics theories, the fuzzy system and Petri net system to describe the behaviour. An industriel application on a tobacco manufacturing plant, where the critical parameter is the weight is presented as an illustration.

Keywords—Petri Net, Manufacturing systems, Performance evaluation, Fuzzy logic, Tolerant system.

I. INTRODUCTION

MANUFACTURING systems and particularly chemical and food industries are based on time constrained processes. More precisely, these processes are performed during certain time windows. Modelling such processes must specify, for every operation, either a minimal duration or an operating duration included in a time window, in order to ensure the product quality. Several studies describe appropriate models based on Timed Petri Nets and are able to model processes with time constraints [1], [10]. In the category of the workshops, the operations have temporal constraints that must be imperatively respected. The violation of these constraints can damage the health of consumers. The constraint violation causes a deterioration of quality or the stop of production.

In many processes, the deciding parameter for the quality and the cost is not time. However, this parameter must strictly belong to some validity intervals. The control must guarantee the fulfilment of these specifications. So a suitable model, rich in analytical properties, is necessary to synthesize the needed controller.

The aim of this paper is to explain the use of Interval Constrained Petri Nets (ICPN) a sub-class of High Level Petri Nets with Abstract Marking (AM-HLPN) [3], [12].

Nabli L. Author is with the National school Engineers of Monastir, Road of Kairouan, Monastir 5000, Tunisia (phone: 0021698503487; fax: 002163492758; e-mail: lotfi.nabli@enim.rnu.tn).

Dhouibi H. Author is with the National school Engineers of Monastir, Road of Kairouan, Monastir 5000, Tunisia (e-mail: hedi.dhouibi@laposte.net)

Collart Dutilleul S. is with the Central school of Lille France, (e-mail: simon.collart@ig2i.fr)

Craye E, is with the Central school of Lille France, (e-mail: etienne.craye@ec-lille.fr)

ICPN allow one to model and guarantee a constraint on any parameter of a manufacturing process. They are applied to a tobacco factory unit where the speed of machines is constant.

We ensure the product quality by verifying weight specifications. The production is optimised by minimizing losses in presence of variation.

The first section introduces ICPN that are a functional abstraction of P-Time Petri Nets [10]. In this way, the extension of structural properties of P-Time PN can be justified under some conditions [4]. A theorem application is extended to our case study and will be illustrated in a practical manner.

The second one describes the manufacturing process of the tobacco factory. In the third one, the Average Dynamics of the Metrics (ADM) will present. The originality of the approach of the follow-up of evolution of the quality of the product resides, on the one hand in the definition of an indication of quality (IQ). On the other hand, in the setting up of a new method based on the Average Dynamics of the Metrics (ADM), that permits the valuation of the quality of the produced to different levels of the proceeding of manufacture.

II. INTERVAL CONSTRAINED PETRI NET

ICPN are introduced to extend the application field of P-Time PN by proceeding to a functional abstraction of the parameter associated places. Naturally, it is logical to find exactly the same mathematical definition of the tool. Nevertheless, if the restriction of the associated parameter to rational numbers is justified when this one is duration, it has no more justification in the case of modelling weight variation or position. Such parameters may take both negative and positive values.

Furthermore, the introduction of a new formalism is an opportunity to review the initial definition. Thus, we present in an unequivocal manner the marking as a multi-set. In the same way the transmission of a quantity conveyed by a token is represented explicitly.

A. Definition

An Interval Constrained Petri Net is an 8-uple $\langle R, M, IS, D, Val, Val_0, X, X_0 \rangle$ where:
- R is a marked Petri Net
- M is an application associating token to places as follow:
  Let $m(p)$ be a place marking.
  Let Va be a set of rational variables.
  Let V be a non empty set of formulas to use a variables

of Va.

Let μV be a multiset defined on V.

$m : P \longrightarrow \mu V$

$p \in P \longrightarrow m(p)$, where $m(p)$ is a place marking we note M the application:

$M : P \longrightarrow N$ (set of positive integer)

$p \longrightarrow Card(m(p))$

- $IS : P \longrightarrow R \cup \{-\infty, +\infty\} \times R \cup \{-\infty, +\infty\}$ defines the intervals associated to places. R is the set of real numbers.

  $p_i \longrightarrow IS_i = [a_i, b_i]$ where $0 \leq a_i \leq b_i$

- D is an application that associates to each pair (place, token) a rational variable q.

  $(q \leq b_i)$. This variable corresponds to a modification of the associated value of a token in a place.

  $D : m(p) \times P \longrightarrow Va$

  $\forall i, 1 \leq i \leq n, n = card(P)$

  Let k be a token, $k \in m(pi)$,

  $k \longrightarrow q_i | a_i \leq q_i \leq b_i$

  where a_i, b_i are rational values fixed by IS.

- X is an application that assigns to each variable a value.

  $X : Va \longrightarrow Q$

  $va \longrightarrow u \in Q$

  X sets q_i.

- X_0 defines the initial values of variables.

- Val associates to each token a formula of values in Q.

  Val is an application of set of the tokens $m(P)$ in V:

  $m(P) \longrightarrow V$

  $(k \in (m(p)) \longrightarrow v \in V$, where k is a given token.

- Val_0 defines to initial formulas associated to tokens.

A token in place p_i is taken into account in transition validations when it has reached a value comprised between a_i and b_i. When the value is greater than b_i the token is said to be dead. Of course, after the firing of an upstream transitions, token are generated in the output places. And their associated values are equal to:

$$Val(k) + q_i(k) \qquad (1)$$

The signification of q and $Val(k)$ are intentionally not fixed in order to provide a general model. Furthermore, a mathematical relationship between the q in different places is supposed to exist. Nevertheless it is not mathematically defined.

As an example with P-time PN there is the following relation $\forall q, dq/dt = 1$. where t represents time.

In ICPN the application X is not mathematically imposed. We will meet, for example, applications where q parameters represent weight variations of cigarettes. In this case, parameter values associated to pairs (place, token) are independent.

B. State definition

A state E is defined by a t-uple $\langle M, D, Val, X, \rangle$ where:

M, and Val are the above defined applications

D and m assign a variable $q_i(k)$ to each token k in a place p_i. A token k of the place pi can take a part in the validation of output transitions if: $q_i(k) \in [a_i, b_i]$, where $[a_i, b_i]$ is the static interval associated to the place p_i. This token k dies when $q_i(k) > b_i$.

X is an application which provides a value for each variable of V. Actually, X defines the real value of each q.

When X is not defined, there exists a way to make the model evolve. Furthermore, some mathematical properties may be outlined. It is the mathematical abstraction.

Definition 2.1: An abstraction on a set $\{x : A$ **.** $psi\}$ may be interpreted as a set a value of the domain associated to the formula A. The A equation using x has to satisfy the constraint ψ.

The parameters defined with constrained variables will be specified, even though X is not defined, using the above definition:

$$A = Val_0(k) + \sum q_j \qquad (2)$$

the j index correspond to the visiting order of places by a given token

$$x = \{q_i\} \qquad (3)$$

$$\psi = \prod (a_i \leq q_i) \wedge (q_i \leq b_i) \qquad (4)$$

The analytical conditions of a transition firing will be considered without fixing the final value of q variables. This approach is really useful to make a system specification.

III. COMPUTING THE NEXT STEP

There are two different ways of reaching a state from a given one. The first solution is to use the evolution of associated variables. The second one is the transition firings. The following two definitions correspond to these two possibilities of evolution.

Definition 2.2: A state $E'(M', D', Val', X')$ is an accessible from another state $E(M, D, Val, X)$ according to associated variable evolution if and only if:

- $M' = M$
- $\forall j\, a\, token\, in\, p_i$:

$$q'_i(j) = q_i(j) + \Delta q_i(j)\, with\, a_i \leq q'_i(j) \leq b_i \qquad (5)$$

where $[a_i, b_i]$ is the static interval of the place p_i. The possibility of reaching $q'_i(j)$ depends generally on the coupling with other q evolutions. This particular aspect is not presented here.

Definition 2.3: A state $E'(M', D', Val', X')$ is an accessible from another state $E(M, D, Val, X)$ by firing transition ti if and only if:

- t_i is validated from E,
- $p \in P, m'(p) = m(p) - Pre(p, t_i) + Post(p, t_i)$ $Pre(p, t_i)$ corresponds to the weight of the output arcs from p to t_i. $Post(p, t_i)$ corresponds to the weight of the input arcs from t_i to p.
- Tokens that remain in the same place keep the same associated value between E and E'.

The newly created tokens take null values for the q counter associated to their new places. The value allocated to the token k' by Val is:

$$Val(k') = Val(k) + q(k) \qquad (6)$$

Where k is a token that is in an input place p_j of t_i and consumed to fire t_i.

The previous firing rule allows computing states and accessibility-relationships. The set of the firing sequences from an initial state specifies the $PN\ behaviour$ as well as sets of accessible markings or validated firing sequences in the case of Autonomous PN

P-time PN: a sub-class of ICPN Actually, P-time PN may be defined as a sub-class of ICPN. It use the following restrictions: The arrival set off IS application is restricted from $Q \cup \{-\infty, +\infty\} \times Q \cup \{-\infty, +\infty\}$ to

$$Q^+ \cup +\infty \times Q^+ \cup +\infty$$

The signification of the parameter associated to the tokens is the age. This restriction respects the $ICPN$ definition. The X application which provides the definition of parameter's evolution is strictly fixed, because it corresponds to the linear time evolution. Let us remark that this function match the monotonous criteria. $ICPN$ may be seen as a functional abstraction of $P - time\ PN$. The time is only a particular value in the domain of possible interpretations of parameters.

Theorem: structural mathematical properties of P-time PN can be extended to $ICPN$ if and only if: $\forall i / 1 \leq i \leq n, n = Card(P), \min i(q_i) \neq -\infty$

Of course, the properties extension implies that the structure and the marking restriction are the same.

Proof: $ICPN$ instances have all the same mathematical model. The only specificity of $P - timePN$ is the restriction of the token associated parameter to positive rational numbers and the definition of the X application.

Consequently, any demonstration which do not use an assumption on X definition and which only use the marking and the net structure will be valid for any instance of $ICPN$. There is only one condition to be fulfilled: the minimal values for the static intervals associated to places must not be equal to minus infinite. Actually, if this minimal value is negative and finite, a simple change of the origin on the axe of parameter values will shift the arrival set of IS intervals into positive values.

Nevertheless, the physical interpretation of the same property in each instance of model will depend on the physical meaning of parameters which are associated to the tokens.

A. Using a theorem of P–time PN

[2] Let $Ct1$ and $Ct2$ be two different adjustments to instantiate associated values to pairs (place, token).

$C2$ is the cycle value for $Ct2$ and $C1$ is the cycle value for $Ct1$.

p_i has an associated validity interval $[ai, bi]$.

We call $q_{i,Ctj}$ the effective value of the staying time of one token during a mono-periodic functioning according to an adjustment Ctj.

$$At_{i,Ctj} = q_{i,Ctj} - a_i \qquad (7)$$

There is a succession of states that respect staying time constraints for all tokens in the net. It allows moving from a state to another within mono-periodic functioning corresponding to Cti to a state that belongs to a mono-periodic functioning corresponding to Ctj. The transition between Cti and Ctj is noted $Ctij$.

dt_{ij} represents the total duration of the transient mode Ct_{ij} [2].

Let SC_i the primary critical structure indexed by i and nc_i the number of tokens in SC_i . Rigorously, this magnitude must be a function of time.

Suppose that our workshop contains only primary critical structures. Call pr the place modelling a constraint within a critical structure SC_r.

Theorem Collart 2002: $(\forall At_{r,Ct1}, \exists C_1 / At_{r,Ct1} > 0) \Rightarrow \exists t_{12}$

$$nt_{12} = [(C_2 - C_1) / \Delta C*] + 1$$

$$dt_{12} = n* \times nt12 \times (C_1 + (\Delta C*.nt12)/2)$$

where

$$\Delta C* = \min_r (At_{r,Ct1} / ncr)$$

$$n* = \max_r (nc_r)$$

$nt12$: represents the number of intermediate steps between the adjustments $Ct1$ and $Ct2$. The intermediate step includes firing all the transitions of the net for $n*$ times.

ncr: is the number of tokens in an elementary circuit or the difference of tokens between a synchronization transition and a parallelism one. It is quite normal in a dynamic system that an event requires multiple resources. The resulting synchronization of resources can be captured by transitions of the type shown in figure 1. Here, t_4 is enabled only when each of P_3 and P_4 receives a token. The arrival of a token into each of the two places could be the result of a possibly complex sequence of operations elsewhere in the rest of the Petri net model. Essentially, transition t1 models the joining operation. Parallel computing is a form of computation in which many instructions are carried out simultaneously, the transitions t_2 and t_3 are concurrent.

Concurrency is an important attribute of system interactions. Note that a necessary condition for transitions to be concurrent is the existence of a forking transition that deposits a token in two or more output places.

The proof was been given in the case of P-Time PN. A functional abstraction allows one to generalize the property to ICPN.

The marking functions that depend on time have fixed initial values, henceforth.

The algorithm allowing the transition from adjustments Ct_1 to another Ct_2 is as follows. φ_{cd} :

$$SC_r = \{p_k, {}^\circ p_k, p_k^\circ\}_{k \in N + \cap [i,j]} \rightarrow SC_{r*} = \{p_k, {}^\circ p_k, p_k^\circ, p_{r*}\}_{k \in N + \cap [i,j]}$$

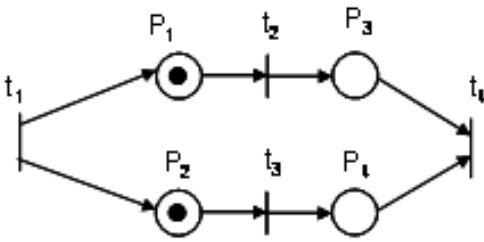where:

$$a_k = \min (q_{k,Ctc}, q_{k,Ct2})$$

Fig. 1. The typical characteristics in PN

$$a_k = \max\left(q_{k,Ctc}, q_{k,Ct2}\right)$$

$$p_{r*}^{\circ} = p_j^{\circ}$$

$$^{\circ}p_{r*} = ^{\circ}p_i$$

$$a_{r*} = \sum_{k=i}^{k=j} q_{k,Ctc} - \sum_{k=i}^{k=j} M(p_k) \times C_d + nc_r \times \Delta C^*$$

$$b_{r*} = \infty$$

Remark: Place pr* models the staying time constraints. It specifies the global variation of the staying time in a specific elementary circuit.

Let R be the expansion of the net into critical circuits SC_r.

The algorithm below [3] is applied in order to compute the set:

$$\sum_{k=i}^{k=j} \Delta q_k$$

i) The first step: construction mode transitions
for $r = 1 \, to \, card|R|$ if

$$\left(\varepsilon \times \sum_{k=i}^{k=j} (q_{Ct2} - q_{Ct*}) \geq \varepsilon \times n_{Cr} \times \Delta C^* \right)$$

then

$$SC_r \leftarrow \varphi_{1*}(SC_r)$$

$$\forall m \in [i,j], \left\{ \begin{array}{l} q_{m,Ct1} < q_{m,Ct2} \\ q_{m,Ct_{n*}} = q_{m,Ct2} \end{array} \right\} \vee \left\{ \begin{array}{l} q_{m,Ct1} > q_{m,Ct2} \\ \Delta q_m = 0 \end{array} \right\}$$

else end for

ii) The second step: Algorithm for calculating firing time in a Strongly Connected Event Graph

An algorithm computes the firing instants in $\varphi_*(G)$ if $C1 < C2$ and in other cases:

• *Case where* $C1 < C2$

A graph G' is associated to a strongly Connected Graph events Related P-time G within a 1-periodic functioning with period equal C. The nodes of G' corresponds to transitions of G.

The arrows of G' are obtained from G's places: two arrows are associated to each place p.

– the first arrow links $^{\circ}p$ to p°, its ponderation is computed according to the next formula :

$$Vp = a_p - C \times m_p \qquad (8)$$

– the second links p to p, its ponderation is evaluated as follows:

$$V'_p = -b_p + C \times m_p \qquad (9)$$

– Choose a transition t_s, and associate $Sts(1) = 0$ to t_s
– Associate to each transition

$$t_u \in T, St_u(1) = \max_{l_{su}} \sum_{p \in l_{su}} v_p$$

– *Other case*

$$t_u \in T, St_u(1) = \min_{l_{su}} \sum_{p \in l_{su}} v'_p$$

where $l_s u$ is an direct elementary path linking s to u. In our case, the return constraint $[a_i, b_i]$ leads us to modify the algorithm [3]. In fact, this one supposes that there is no time constraint in places modelling robot comeback. The interval allocated to such a place looks like $[a_i, \infty]$.
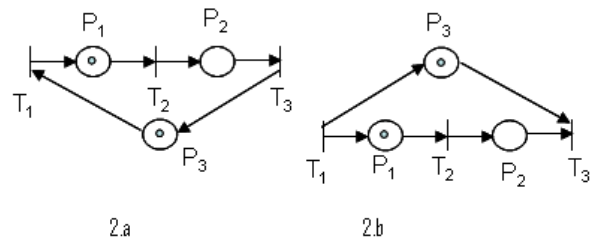
B. Calculating Waiting times

Fig. 2. Waiting time

Consider the cases described in figure 2 a and b. Waiting times can be computed in two cases:

Case 1: Delay or C cycle time extension. A token reaches a transition T_e early. The problem is how to manage a part (P_1) that arrives early while a second part (P_2), to fulfil production, is not yet present. In this case, we check if there is a waiting margin for this part in P_2. These margins are computed as follows:

$Ati = q_i - a_i$: for the "a"$circuit$ $Ati = b_i - q_i$: for the "b"$circuit$

Case 2: Advance or C cycle time reduction. A token reaches a transition T_e late. In the same way we look for an available margin of this token, in order to make a synchronization with a token already existing in place P. This margin is given by the following relationships:

$Ati = b_i - q_i$: for the "a"$circuit$ $Ati = q_i - a_i$: for the "b"$circuit$

Remark: When there is no token difference within places, this problem do not appear.

IV. Application to regulating the weight of cigarettes

The previous section ends with a theorem useful in the regulation of discrete processes working under time constraints. In this section, we extend the concept of p-time Petri nets to the interval constrained Petri nets. We try to apply our proposed net structure in the field of quality control of tobacco production, where time constraints cannot be ignored when discrete-event-driven models of such systems are established.

Definition 1: An ICPN model "$WeightValue$" is an ICPN model where the value associated with the couple $(place, token)$ is interpreted as a $weight$.

Of course, it is not a question here to introduce a new sub-class of ICPN for every interpretation.

This tool ICPN "$WeightValue$" allows one to model a manufacturing process where the weight of the produced item must belong to a given interval.

This is, for example, the case of cigarette making workshop of tobacco industry described below.

As an example, we will use a proof that gives analytical conditions for the evolution from a cyclic functioning point to another within a P-time PN. After the presentation and modelling of the process, we will discuss the new significance of this theorem in the case of the ICPN "$WeightValue$" and its ensuing applications.

A. Workshop design

One of manufacturing systems where the respect of a produced item's weight arises is a cigarette making workshop. The process, as shown in figure 3, makes a regular and homogenous tobacco pudding (endless cigarette): A beam of tobacco is enveloped by cigarette paper by means of an adhesive. The resulting pudding is cut up into segments corresponding to one cigarette in order to obtain a rough consumed unit (cigarette without filter).

Within this process, a weight interval constraint must be respected.

In fact, from a quality point of view a too heavy cigarette is difficult to draw and a too light one does not satisfy requirements.

The production of cigarettes consists of three steps as illustrated in figure 3:

- Preparation of a tobacco cut beam that will be setting to obtain a module (**m**).
- Forming of a pudding with density (**d**) by enveloping the beam with cigarette paper.
- Cutting up the pudding into (l) long rough consumed units. Henceforth, such unit is called simply cigarettes.

A cigarette may be compared to a cylinder with diameter m and length l. Thus, the weight of a cigarette can be expressed by the following relationship:

$$P = \Pi/4 \times m^2 \times l \times d \qquad (10)$$

where :

- **m:** cigarette's module in mm with $m \in [m_{min}, m_{max}]$;
- **l:** cigarette's length in mm with $l \in [l_{min}, l_{max}]$;

- **d:** density in g/mm3 with $d \in [d_{min}, d_{max}]$;

Note that these parameters (weight, module, length and density) are related. Obviously, the variation of one of these parameters provides a variation of the weight. When it is outside the validity range, the production has to be rejected or a machine blocking.

Our objective is to make sure that the permitted tolerance concerning cigarettes weight will be respected by controlling m, l and d parameters. It must belong to a predefined interval. The aim of the controller is to maintain the weight specification by changing the setting m, l and d parameters, whereas they have to remain in a validity interval:

$$P_min \leq \Pi/4 \times m^2 \times l \times d \leq P_max \qquad (11)$$
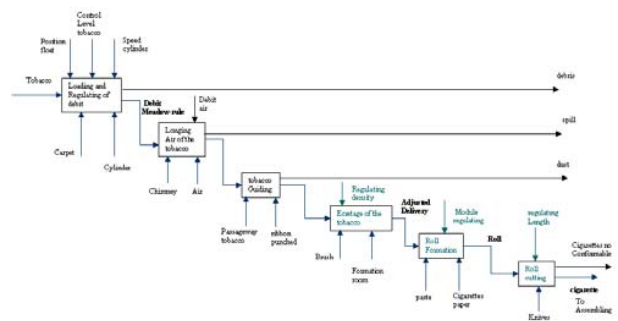
B. Linearization around a reference state

Fig. 3. Process of manufacturing system

we only proceed to a first or second order development). The four parameters taking part in equation (11) are not perfectly constants. They change around a medium position which corresponds to the stable state of the systems. Then we can assimilate the equation (11) to a linear relationship by proceeding to one first order limited development and we can write growth of this function in the following linear form:

$$\Delta P \leq K_1 \times \Delta d + K_2 \times \Delta m + K_3 \times \Delta l \qquad (12)$$

with:

- $K_1 = \frac{\Pi}{4} \times l_0 \times m_0^2$
- $K_2 = \frac{\Pi}{2} \times l_0 \times m_0 \times d_0$
- $K_3 = \frac{\Pi}{4} \times d_0 \times m_0^2$

$\Delta P \in [P_{\min}, P_{\max}]$

p_0, m_0, c_0 and l_0, respectively the values targets of the parameters p, m, c and l.

C. Construction of model "weight value"

The equation (12) can be represented by the ICPN shown in fig. 3. It called "$WeightValue$".

The PN of figure (4) represents the equation (12) that models weight variations concerning the three parameters: density, module and length. Thus, the medium variation of circuits is not equal for all parameters. But, if we balance arrow weights, we could have obtained the same medium value
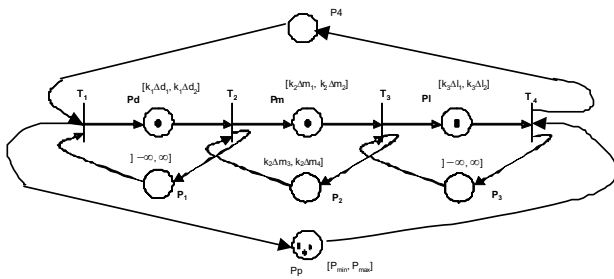
Fig. 4.   ICPN model "$weight value$"

for all circuits. This system's invariant characterizes the weight conservation within the process.

The model in figure 4 consists of:

- 8 places $P_c, P_m, P_l, P_p, P_1, P_2, P_3 and P_4$
- 4 Transitions T_1, T_2, T_3 and T_4
- 5 elementary circuits:
- $\gamma_1 = T_1, P_d, T_2, P_1$
- $\gamma_2 = T2, P_m, T_3, P_2$
- $\gamma_3 = T_3, P_l, T_4, P_3$
- $\gamma_4 = T_1, P_p, T_4, P_4$
- $\gamma_5 = T_1, P_d, T_2, P_m, T_3, P_1, T_4, P_4$

Places P_1 and P_3 are used to limit at one the number of tokens for a specific operation. As they have not other uses we will associate to them the interval $(\infty-, +\infty)$.

Place P_2 models the constraint characterizing module variation.

The production process corresponding to places P_d and P_m (density and module) is continuous. It is considered as a discrete one during our study. In fact, module and density measurements are done through pecimens. We consider that twenty cigarettes can give us good representative and reliable information.

Density is subject to hazardous variations. Its distribution follows the normal distribution (based on production data). We will then compensate these lapses in acting on the module and length that are adjustable and depend on the state system.

However, this regulation should include a coupling system that affects its dynamic behavior.

D. Condition theorem verifying

The cigarette production cell can be decomposed into critical structures. In fact, the succession of transformation operations constitutes a sequence. Elsewhere, limitations of physical stocks provoked by cyclicity constraints are modelled by the presence of combacks within the process. This leads to the appearance of elementary circuits.

Finally, constraints that concern weight are represented by a place that is parallel to the operating process model. There are, then, pair of paths that start with parallel transitions and end with a synchronization one. Here again, there is a conformity with the definition of primary critical structures. The considered workshop meets the application conditions of the theorem emitted previously.

TABLE I
COMPARISON OF PARAMETERS P-TIME PN/ICPN MODEL

Parameter	P-Time PN signification	ICPN model
C	Time Cycle	Weighing cycle for a piece
ΔC	Variation of time cycle	Variation of weight per cycle for a piece (compared to a reference)
Q	Effective time passed in a place	Weight added in this place
ΔQ	Variation of effective time	Variation of the added weight in place compared to a reference
a_i	Lower bound indicates the minimum time needed to execute the operation	Lower bound indicates the minimum weight added otherwise the quality of product is deteriorated
b_i	The upper bound fixes the maximum time to not exceed	Upper bound indicates the maximum weight added otherwise the quality of product is deteriorated
M	Product, resource , constraint	UProduct, resource, constraint.

In order to illustrate this theory, consider the model "$WeightValue$" in figure 4. It models weight variation of a cigarette according to equation (12).

Mathematically, time PN and any $ICPN$ (for example the $ICPN$ "$WeightValue$") have the same properties. However, the physical interpretation that must be given to the model is completely different.

Table I summarizes significance of different parameters that take part in both time PN and the $ICPN$ "$WeightValue$".

The effective value of a parameter can be calculated with polynomial algorithms. This can be done because the above algorithm is only based pon the structural properties of P-time Petri Net. In this case, it was shown that, under some particular assumptions, the property may be extended to $ICPN$ [2]. In fact, the medium weight of cigarettes must be as near as possible from the target weight.

As it is impossible to check all cigarettes for high speed production, we proceed by selecting successively a quantity of cigarettes. Measurements are taken via this specimen in order to compute medium weight value and standard deviation **??**.

When a deviation is detected, we look after a regulation that is able to manage the new value of the density and then the weight.

Figure 5 presents a basic functional block allowing to regulate weight: the micro-onde capture head, the weight regulation and the adjustment of tobacco cordon.

- The Microonde capture head gives metrological values to the weight regulation module.
- Le P calculator determines the current weight of cigarettes and evaluates the production quantity. The cigarette weight is automatically controlled. In case of variations (A) compared to order (B) the weight controller WT react by computing a new order (D) which will affect the Ecreteur Disks. The positioning system POS performs the corresponding weight correction. If the real weight and the expected one do not coincide the regulation module computes a new position for a shop
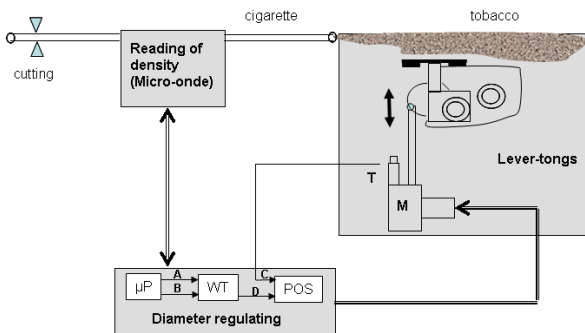
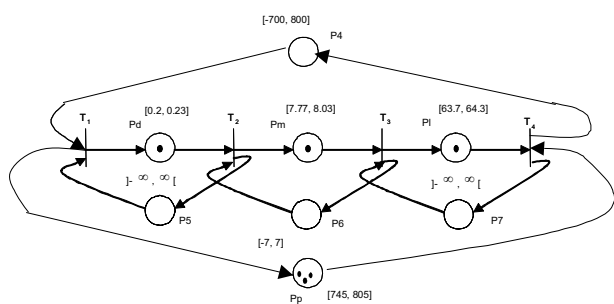Fig. 5.   Structure of the regulatory system

Fig. 6.   ICPN target value

system. When the linear Tranducer T indicates that this one has reached the new position, a new comparison between the effective weight and the expected one is carried out.

A reversing relay controls the rotation sign of servomotor M in order to Ecreteur Disks (Fig. 5 up or down according to the weight value in order to decrease or increase tobacco amount.

Deviation with target value is computed by studying a specimen of 20 cigarettes.

Under these conditions, the application of the theorem presented previously is exactly identical. So, there is a noticeable difference between the P-time model and the ICPN model "value weight" of our example: the "value weight" uses a differential of the first order around a reference point that allows us to linearize the pattern. Indeed, it is clear that in general, the calculation of cycle time in ICPN model is not linear. In this way, one can quantify the change in weight that occurs at different positions. With the help of this property one could therefore move within the space of valid solutions. Figure 6 represents the model "$value weight$" trucking (Fig. 4) with intervals of validity of each parameter.

Suppose that the initial values of parameters $module$, $length$ and $weight$ are respectively:

$m = 8$ mm, $l = 64$ mm and $P = 770$ mm.

The density of the sample production is $d = 0.21$.

Our objective is to adjust the weight to $P = 740$ g. Intermediary regulations allow to reach this objective as given in table II

In this table, $A_t p$ and $A_t m$ are respectively available mar-

TABLE II
SETTINGS INTERMEDIARIES TO A TARGET VALUES FOR *d=0.21*

Step	1	2	3	4	5	6	7	8
m	8	7.98	7.95	7.93	7.9	7.9	7.9	7.9
l	64	64	64	64	64	63.8	63.5	63
P	770	768	762	758	752	750	746	740
$A_t p$	30	28	22	18	12	10	6	0
$A_t m$	2	2	2	2	2	2	2	2

TABLE III
SETTINGS INTERMEDIARIES TO COMPENSATE A DENSITY FLUCTUATION
FOR *d=0.22*

1	2	3	4	5	6	7	8	9	10	11
7.9	7.93	7.95	7.98	8	8	8	8.05	8.08	8.1	8.1
63.5	63.5	63.5	63.5	63.8	64	64	64	64	64.5	64.5
684	690	693	698	702	705	707	716	721	725	**730**
47	41	38	33	29	26	24	15	10	6	0
2	2	2	2	2	2	2	2	2	2	2

gins in places Pp and Pm used to respect the product quality. These margins are specific to transitory regulations.

Suppose now that the density value is derived to take a new value **d = 0.22**. This obliges one to proceed to the other regulations in order to compensate this deviation and reach the target weight. Table III gives intermediate adjustments and the weight value we are able to reach.

Consider now quantity variations in comparison to medium values representing targets values. Then we consider that values described in Fig. 4 are such that every step of the Q_i are the values recorded in table IV.

For example, for step 1, $m = 8mm$ with constraint on the module such as $m = [7.9, 8.1]$, we find $m_1 = [-0.1, 0.1]$. A similar reasoning on all places for the first step gives the first column of table III. We proceed in the same way for the remaining steps the Table IV is obtained.

E. Using ADM for control maintaining the quality

The method of the Average Dynamics of the Metrics (ADM) [8] is based on three stages (fig. 7):

- **Fuzzyfication**: the stage of consists in converting them in vague subsets modelling the human expertise.
  To prepare the stage of numeric inference, we associate to every variable Z a function of classification Cl, allowing

TABLE IV
VARIATION OF PARAMETERS RELATIVE TO THE TARGET VALUES FOR
d=0.23

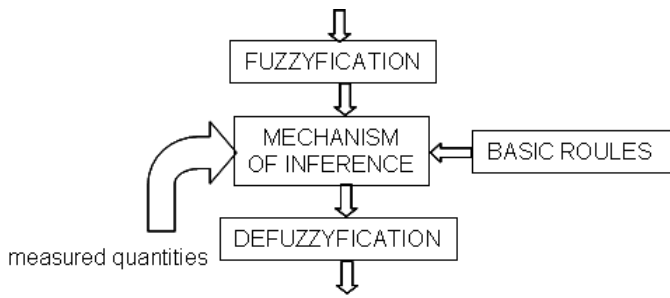| Step	Δm	Δl	ΔP
1	[-0.1, 0.1]	[-1, 0]	[-50, 40]
2	[-0.08, 0.12]	[-1, 0]	[-48, 52]
3	[-0.05, 0.15]	[-1, 0]	[-42, 48]
4	[-0.06, 0.17]	[-1, 0]	[-38, 52]
5	[0, 0.02]	[-1, 0]	[-32, 58]
6	[0, 0.02]	[-0.8, 0.2]	[-30, 60]

Fig. 7.   Stages of method ADM

classifying every measure of the variable in one of the vague subsets defined by the fuzzification. Formally we define Cl in the following way: $Cl :\rightarrow [1$

$cdotsm(Z)], Z_k \rightarrow [Cl(Z_k]$

with: $D = [Zmin...Zmax]$ the domain of definition of the variable $Z, m(Z)$ the vague number of set define for the variable Z, a value of the variable Z and w the number of the class to which belongs the value.

The function of classification Cl is defined so as to model from the point of view of the expert, the nature of the evolution of the variable Z. So, if a variable has a monotonous evolution, the function of classification is going to classify in a monotonous way the vague subsets.

- **Mechanism of inference**: The stage consists on definition of certain number of rules of inferences allowing coding the human expertise under a vague shape.

  Generally, the coding is not exhaustive and the expert retains a number of rules which well seem to him to model the problem. These rules contribute to the calculation of the exits according to the reserved mechanism of inference.

  Within the framework of the ADM, it consists in realizing a projection of the space of the variables of entrance (entry) in the space of the variables of exit (release). To realize this projection we introduce the notion of factor of level-headedness b_j associated to a variable of entry X_j:

$b_j : D_j \rightarrow \Re \ X_j \rightarrow b_j(X_j)$

These factors of level-headedness have a double meaning. On one hand, they serve for quantifying the influence of a variable of entry to the determination of the value of a variable of exit. On the other hand, they allow to normer the space of representation of the variables of exit considering the classification realized on the vague subsets of every variable. Indeed, we have to take into account the fact that the number of classes associated to every variable of exit is not necessarily the same that that of the variables of entry. According to, needs the factors of level-headedness go allow to dilate or to reduce the space of a variable of exit to term of number of classes defined by Cl.

Knowing that the expertise allowed expressing the knowledge on the process, we suggest formalizing this knowledge by a function of numeric projection P_k, defined for

every variable of exit Y_k. She allows determining the class of membership of the value of a variable of exit Y_k knowing the classes of membership of the values of the variables of entries X_j.

$P_k : D_{(yk)} \rightarrow [1 \cdots m(Y_k)] \ Y_k \rightarrow P_k(Y_k)$

With: $P_k(Y_k) = E \left(0.5 + \sum_{j=1}^{n} b_j^k(X_j)^*Cl_j(X_j)) \right)$ With:

DY_k the domain of definition of the variable of exit Y_k, $M(Y_k)$ counts him of vague subsets associated to the variable Y_k, $Bk_j(X_j)$ the factor of level-headedness of the variable X_j concerning the determination of Y_k, $Cl_j(X_j)$ the value of the class associated to the common(current) value of the variable X_j.

The sum of products $bk_j(X_j) * Cl_j(X_j)$ allows to determine a value approached the class associated to Y_k. The addition of the coefficient 0.5 and the rounded off by the calculation of the whole value, allows us to determine the closest integer for the class of exit. The coefficients $b_kj(X_j)$ is defined in a way that the definite value is included between 1 and $m(Y_k)$ considering the current value of every variable of entry X_j.

Knowing that the expertise allowed expressing the knowledge on the process, we suggest formalizing this knowledge by a function of numeric projection P_k, defined for every variable of exit Y_k. She allows determining the class of membership of the value of a variable of exit Y_k knowing the classes of membership of the values of the variables of entries X_j.

- **Defuzzufication**: Within the framework of the ADM, the $dfuzzification$ consists in calculating the numerical value associated to a variable of exit. This calculation is elaborated from the knowledge of its class of membership defined by the numeric inference and from the values of the functions of membership relative to the variables of entries. To formalize this calculation, we associate to every variable Y_k a metrics defined by the function f_k:

$$f_k : X_*^n D_{(}Yk) \rightarrow [0,1] \qquad (13)$$

$(X, Y_k) \rightarrow \mu_k(Y_k)$
with

$$\mu_k(Y_k) = f_k \left((\mu_n(X_n) \cdots (\mu_1(X_1), \left(b_j^k(X_n) \cdots \left(b_j^k(X_1) \right) \right) \right)$$
$$(14)$$

with $X = [X1, Xj, Xn]^T$ the vector entry defines on $X_n \ K(Y_k)$ the degree of membership of the current value of the exit Y_k, $J(X_j$ the degree of membership of the current value of the entry X_j, $Bk_j(X_j)$ the factor of level-headedness of the variable X_j concerning the determination of Y_k.

Once class of exits and the degree of membership of the exit known, it is necessary to apply a method of $dfuzzification$ adapted to the nature monotonous or not of the function of classification associated to the variable of exit. It exists in the literature of numerous methods of $dfuzzification$ as the method of the *centre* of gravity. We name function of partial membership, the function defining the curve wraps with a given class. If the function(of partial membership defining
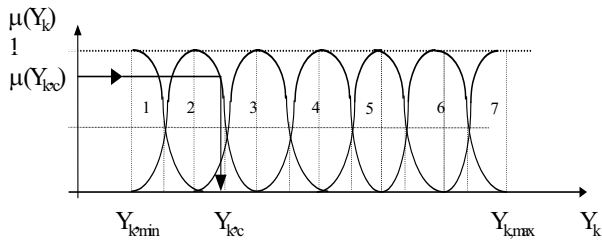
Fig. 8. $Dfuzzification$ by projection on the Y_K axis Framework of a classification
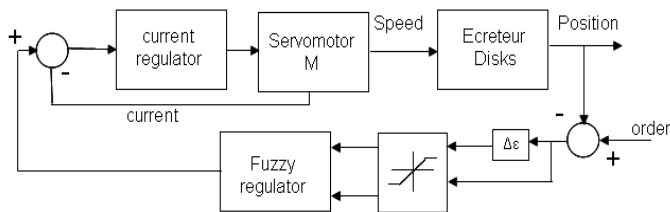

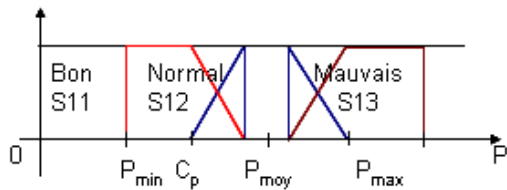Fig. 10. Structure of regulatory including Fuzzy logic
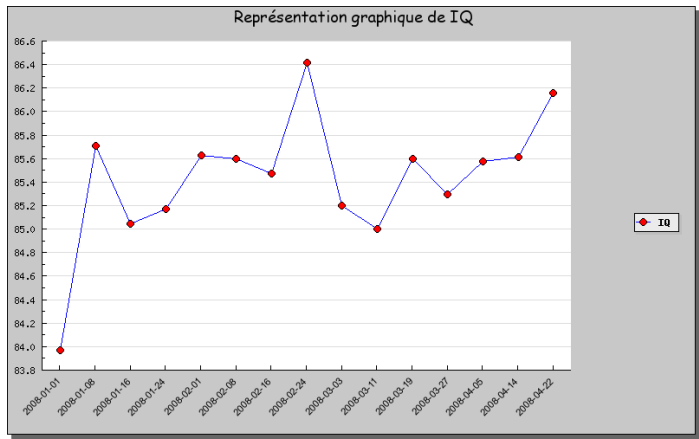

Fig. 9. Trapezoidal possibility distribution

Fig. 11. Variation of Quality indication by ADM

the class of exit in a monotonous evolution, a method of $dfuzzification$ consists in throwing(planning) on the X axis the point of the curve defining the class with the right-hand side stemming from the orderly $k(Y_kc)$ (Fig. 8).

The originality of the approach of the follow-up of evolution of the quality of the product resides, on the one hand in the definition of an indication of quality (IQ). On the other hand, in the setting up of a new method based on the Average Dynamics of the Metrics (ADM), that permits the valuation of the quality of the produced to different levels of the proceeding of manufacture.

In our case, the quality indication IQ representing the variations of three quality parameters is quantified by the exploitation of the method ADM. This method is applied from the same functions of membership allowing the linguistic qualification of the partial indicators of the quality (d, m, l) (Fig. 9).

P_{max} the maximum allowable weight P_{moy} the average weight $P_{min} =$ the minimum weight possible $varepsilon = Tolerance of weight$

In the objective to put in evidence the effect of the number of vague classes on the precision of the estimation of the quality of the product used in the manufacturing process. They allow making the quality control of the product and the follow-up of evolution of the system of production.

In our case, the parameters measured are the reel position of the $Ecreteur$ Disks and error. Both values are use by the regulator whose structure is shown in figure 10.

TABLE V
: LOGIC OF MODERATION

Components	d	m	l
Moderation	50%	35%	15%

The MDM on the other hand allows a continuous follow-up of the evolution of the quality indication (Fig. 11). This evolution continues allows us to fix thresholds of detection within a class of membership of the quality indication. This one thus allows us to discover in most rather damages.

F. Interpretation of results

A brief interpretation of weight constraint is given here. As the graph representing the system is strongly connected, the weighting associated to an elementary circuit corresponds to the value in mg relative to the target weight. Remember that we have proceeded to a first order limited development around this reference.

Theorem introduced in this section gives a condition to change cyclic instruction. Considering the slowly drifts of the process they can be anticipated. It is exactly the case of our study, if we consider variations of tobacco density that must be compensated by length or module corrections.

The theorem gives a guarantee concerning system reactivity: a maximal number of cycles is given to move from an adjustment to another. It will be possible, then, to compute a new instantiation of parameters (able to manage the new value of the density) and to move towards this new adjustment.

During this procedure, the respect of constraints is ensured. Finally, as constraint violations lead to reject some quantity

of products present on machines, we have provided analytical tools able to reduce rejections. Within the studied production cell, all other machines work on the same speed of the critical machine. By reducing variations, production rates are proportionally brought up.

V. CONCLUSION

In this paper, we attempt mixing two different characteristics theories: The method of the Average Dynamics of the Metrics (ADM) and the Interval Constrained Petri Net.

The ICPN model "Weight Value" concerns processes where the conformity of the finished product depends on the value of the weight by a produced unit. This value must belong to a certain validity interval. Outside this interval product is considered as rejection. To improve production performance within such a process, the control of the quality constraint must be able to adjust fluctuations that affect the system's entries. This adjustment has to be done through intermediate regulations that do not alter specifications. Regulations concern parameters that influence directly the weight. In our case parameters are density, module and length.

The proposed approach is portable since we have allowed attention to enable derivation of the ICPN introduced in this paper. Those PN are themselves a restriction of Abstract Marking PN that are clearly defined next to HLPN [3]. The functional abstraction made within ICPN allows to take into account varied linear constraints

One must note that an adjustment near maximal or minimal bounds might increase rejections rates in the case of noised entries. A multicriteria evaluation, in the context of workshop presenting interval validity constraints, could resolve such a problem.

ACKNOWLEDGMENT

REFERENCES

[1] S. Calvez, P. Aygalinc, and W. Khansa , *P-time PN for manufacturing systems with staying times constraints* LaTeX, . IFAC Congress, Belfort, France, pp. 495-500, May 1997

[2] Collart Dutilleul S., Denat J.P and Chetouane F , *Robust Control of Electroplating Lines* LaTeX,. IEEE Conference- Hammamet October 2002, pp. 195-204 .

[3] Collart Dutilleul S., H. Dhouibi and E.Craye , *Internal Robustness of Discret Event System with interval constraints in repetitive functioning mode* LaTeX, Wodes 96. ACS'2003 conference, Miedzyzdroje Poland, pp. 353–361, 2003

[4] Collart Dutilleul S., H. Dhouibi and E.Craye , *Tolerance analysis approach with interval constrained Petri nets* LaTeX, . European Simulation and Modelling Conference, Paris, pp. 265-272

[5] David R. and H. Alla , *Petri Nets and Grafcet : Tools for Modelling Discrete Event Systems* LaTeX,. Prentice-Hall.

[6] H. Dhouibi, S. Collart Dutilleul, E. Craye and L. Nabli , *Computing Intervals Constrained Petri Nets: a tobacco manufacturing application'* LaTeX, Wodes 96. IMACS conference, Paris, pp. 440–446, 2005.

[7] *International Standard Ballot of ISO/IEC 15909-1 (High-level Petri Nets)* LaTeX, .

[8] L. Nabli., K. Ouni, *The indirect supervision of a system of production by the Principal Components Analysis and the Average Dynamics of the Metrics* LaTeX, . International Review of Automatic Control (IREACO), vol.1 n 4. 2008.

[9] Wang Juacun *TTimed Petri Nets: Theory and Application* LaTeX, . Kluwer, Boston, MA, 1998.

[10] W. Khansa, J.P. Denat and S. Collart-Dutilleul , *TP-Time Petri Nets for Manufacturing Systems* LaTeX, . Wodes'96, Edinburgh UK, August 19-21, pp. 94-102, International Workshop on Discrete Event Systems.

[11] Yim P., Lefort A. and Hebrard A , *a constraints based tool for the hypernets* LaTeX, . 7th IEEE Int. Conf. on "Tools with Artificial Intelligence", ICTAI'95, Washington (U.S.A.), pp334-337, November 1995.

[12] P. Yim, A. Lefort, and Hebrard , *System Modelling with Hypernets* LaTeX, . EETFA'96 IEEE Conferences, pp 37–47, Paris, 1996.

Lotfi NABLI received his Mastery of Sciences and DEA from ENSET-Tunis-Tunisia in 1989 and 1991 respectively. In 2000, he obtained his doctorate degree in Industrial automation: Automatic and Industrial computing from University of the sciences and the technologies of Lille France. He is currently Assistant professor of Electrical Engineering at National School of Engineers of Monastir (ENIM)-Tunisia and a Master of conference candidate. His research interests include Modeling, Intelligent Control and Monitoring and command Manufactory systems.

Hedi Dhouibi received his Engineer degree at maintenance and DEA at National School of Engineering, University of Center, Tunisia in 1997 and 1999 respectively. In 2005, he obtained his doctorate degree in Industrial automation: Automatic and Industrial computing from University of the sciences and the technologies of Lille France. He is currently Assistant professor of Electrical Engineering at University of the sciences of Gabès Tunisia. His research interests include Modeling, Intelligente Control and Monitoring and command Manufactory systems.